(فاطمه شعبانی)

تست ۲۳

1)

$\hookrightarrow$ if $x = \frac{\pi}{4} \to \underbrace{\tan^2(\frac{\pi}{4})}_{1} + \underbrace{\sqrt{2}\cos(\frac{\pi}{4})}_{\sqrt{2}\times\frac{\sqrt{2}}{2} = 1} = \underline{2}$

$\hookrightarrow g(\frac{\pi}{4}) = f(2) \to g'(\frac{\pi}{4}) = f' \to$

$g'(\frac{\pi}{4}) = \underbrace{(2 \times \tan x \times (1+\tan^2 x)) + (-\sqrt{2}\sin x)}_{} \times f'(2) \Rightarrow$

$x = \frac{\pi}{4} \to \underbrace{2\times 1 \times 2}_{4} + \underbrace{-\sqrt{2}\times\frac{\sqrt{2}}{2}}_{-1} = 3$

$\hookrightarrow \sqrt{3} = 3 \times f'(2) \to \boxed{f'(2) = \frac{\sqrt{3}}{3}}$

۴) $f(x) = \dfrac{8+\cos^3 x}{4-\cos^2 x}$, $g(x) = \dfrac{2}{2-\cos x}$ $\rightarrow f'(\frac{7\pi}{6}) - 2g'(\frac{7\pi}{6})$ کدام

$(f-2g)'(\frac{7\pi}{6}) = ?$

$* \dfrac{(2+\cancel{\cos x})(4-2\cos x+\cos^2 x)}{(2-\cos x)(\cancel{2+\cos x})} = \dfrac{4-2\cos x+\cos^2 x}{2-\cos x}$

$\rightarrow \dfrac{\cancel{4}-2\cos x+\cos^2 x-\cancel{4}}{2-\cos x} = \dfrac{\cos x(\cancel{\cos x-2})}{\cancel{2-\cos x}} = -\cos x$

$\rightarrow (-\cos x)' = \sin(x) = \sin(\frac{7\pi}{6}) = \boxed{-\frac{1}{2}}$

$*$ ۳) $|4x-3|\sqrt{ax}$ $\quad x=\frac{3}{4} \rightarrow 2\sqrt{6}$

$\frac{3}{4}$: $- \mid +$

حد مشتق راست $\rightarrow x=\frac{3}{4} \rightarrow (4x-3)\sqrt{ax}$ $\quad$ (I)

مشتق چپ $\rightarrow x=\frac{3}{4} \rightarrow (3-4x)\sqrt{ax}$ $\quad$ (II)

$\rightarrow (I)' = 4\sqrt{ax} + \dfrac{a}{2\sqrt{ax}}\underbrace{(4x-3)}_{0} \rightarrow 4\sqrt{\frac{3}{4}a}$

$\rightarrow (II)' = -4\sqrt{ax} + \dfrac{a}{2\sqrt{ax}}\underbrace{(3-4x)}_{0} \rightarrow -4\sqrt{\frac{3}{4}a}$

$\sqrt{\frac{3}{4}a}\times 4 - (-4\sqrt{\frac{3}{4}a}) = 8\sqrt{\frac{3}{4}a} = 2\sqrt{6} \rightarrow \sqrt{\frac{3}{4}a} = \sqrt{\frac{6}{16}} = \sqrt{\frac{3}{8}}$

$\rightarrow \frac{3}{4}a = \frac{3}{8} \rightarrow \boxed{a = \frac{1}{2}}$

۴) $y+ax=2 \rightarrow x=4 \rightarrow$ f بر مماس

$f(4)+f'(4)=1 \longrightarrow f'(4)=?$

$y=-ax+2 \longrightarrow f'(4)=-a$

$\hookrightarrow y=f(4)=-4a+2$

$-4a+2-a=-1$

$-5a=-3 \rightarrow a=\frac{3}{5}$

$\hookrightarrow f'(4)=-a=\boxed{-\frac{3}{5}}$

۵) $7y-x=5 \rightarrow \frac{ax-1}{3x+1} \rightarrow a=?$

مماس آنها ← $\Delta=0$

$\hookrightarrow \frac{ax-1}{3x+1}=\frac{5+x}{7} \longrightarrow 7ax-7=3x^2+16x+5$

$\rightarrow 3x^2+(-7a+16)x+12=0 \rightarrow \underset{\Delta}{(16-7a)^2-144=0}$

$\rightarrow |16-7a|=12 \nearrow a=\frac{4}{7} \quad \searrow a=4 \longrightarrow$ با بررسی ۱ یا ۲!

$a=4 \rightarrow 3x^2-12x+12=0 \rightarrow 3(x-2)^2=0 \rightarrow x=2$ ✓

$\hookrightarrow \boxed{a=4}$

۶) مماس بر محور x $\rightarrow 3x^2+a=0 \rightarrow 12+a=0 \rightarrow \boxed{a=-12}$

$\hookrightarrow x^3+ax-b=0 \rightarrow \underbrace{8-24}_{-16}-b=0 \quad \boxed{b=-16}$

$\rightarrow b-a=-16+12=\boxed{-4}$

۷) $f(x) = \sin^n(x^۲)$ $\quad \lim \dfrac{f(x)\,f'(x)}{(1-\cos x)^m} = ۳۲\sqrt{۲}$

$۲m+n \to ?$

$\to$ در $\frac{0}{0}$ $\to$ [illegible] $\longrightarrow$

۸) $\to y = \dfrac{\sqrt{x}+1}{\sqrt{x}-1} \to y\sqrt{x} - y = \sqrt{x}+1$

$y\sqrt{x} - \sqrt{x} = y+1 \to \sqrt{x}(y-1) = y+1$

$\to \sqrt{x} = \dfrac{y+1}{y-1} \to x = \left(\dfrac{y+1}{y-1}\right)^۲ \xrightarrow[f^{-1}]{} y = \left(\dfrac{x+1}{x-1}\right)^۲$

$\to y = \dfrac{x^۲+۲x+1}{x^۲-۲x+1} \xrightarrow{x=۲} y' \Rightarrow \dfrac{(۲x+۲)(x^۲-۲x+1) - (۲x-۲)(x^۲+۲x+1)}{(x^۲-۲x+1)^۲}$

$= \dfrac{(۶\times1) - (۲\times۹)}{1} = ۶-۱۸ = \boxed{-۱۲}$

9) $f(x) = (x^2+1)^3(ax+1) \to [-1,0] \to 11$

$\hookrightarrow \dfrac{f(0)-f(-1)}{0-(-1)} = -11 \to \dfrac{1-(8\times\overbrace{(-a+1)}^{-8a+8})}{1} = -11$

$\to \underbrace{1+8a-8}_{8a-7} = -11 \to 8a = -4 \to a = -\frac{1}{2}$

$\to x = -2a = -2\times -\frac{1}{2} = \underline{1}$

$\to f'(1) \Rightarrow (x^2+1)^3(-\frac{1}{2}x+1) \to$

$3(x^2+1)^2(2x)(-\frac{1}{2}x+1) + (-\frac{1}{2})(x^2+1)^3 =$

$\underbrace{3\times 4\times \cancel{2}\times\frac{1}{\cancel{2}}}_{12} + \underbrace{(-\frac{1}{\cancel{2}}\times \cancel{8})}_{-4} = \boxed{8}$

10) $\to \dfrac{f(\frac{\pi}{2})-f(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}} = \dfrac{-1-(0)}{\frac{\pi}{4}} = \dfrac{-4}{\pi}$

$y = \sin x \cos 2x$

$f(\frac{\pi}{2}) = \underbrace{\sin x}_{1}\underbrace{\cos 2x}_{-1} = -1 \qquad f(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}\times 0 = 0$

$\to$ $y = \sin^2 x - \cos^2 x \to f(\frac{\pi}{2}) = 1-0 = 1$

$f(\frac{\pi}{4}) = 0$

$(\sin x = \cos x)$

$\Big\} \to \dfrac{1-0}{\frac{\pi}{4}} = \dfrac{4}{\pi}$

$\to \dfrac{\frac{-4}{\pi}}{\frac{4}{\pi}} = \boxed{-1}$