شیب خط مماس $= \frac{\Delta y}{\Delta x} = \frac{5-1}{3-0} = \frac{4}{3}$

۱- خط $y = ax+b \xrightarrow{x=1} y = a+b = 0 \Rightarrow y = ax - a$ $\quad y = 3a - a = 5 \Rightarrow a = \frac{5}{2} \Rightarrow f'(2) = \frac{5}{2}$ ⑤

۲- $y = ax+b \quad a = \frac{2-1}{2-(-1)} = \frac{1}{3} \Rightarrow y = \frac{1}{3}x - 1 + \frac{4}{3} = 1 \Rightarrow y = \frac{1}{3}x + \frac{4}{3}$ ①

$f'(x) = \frac{a}{2\sqrt{ax-1}} = \frac{1}{3} \Rightarrow 3a = 2\sqrt{ax-1} \Rightarrow 9a^2 = 4ax - 4$

$\Rightarrow 9a^2 - 4ax + 4 = 0 \Rightarrow (3a-2)^2 = 9a^2 + 4 - 12a \Rightarrow x = 3,\ a = \frac{2}{3}$ ×

$\Rightarrow 1 + (\frac{8}{9} - \frac{6}{9})3 + \frac{25}{9} = 1 + \frac{36}{9} \neq 0$

$\Rightarrow (3a+2)^2 = 9a^2 + 4 + 12a \Rightarrow x = -3,\ a = -\frac{2}{3}$ ×

$\Rightarrow 1 + (\frac{8}{9} + \frac{6}{9})(-3) + \frac{25}{9} = \frac{34}{9} - \frac{42}{9} \neq 0$

$\frac{1}{3}x + \frac{4}{3} = \sqrt{ax-1}$

$\Rightarrow \frac{1}{9}x^2 + \frac{16}{9} + \frac{8}{9}x = ax - \frac{9}{9}$

$\Rightarrow \frac{1}{9}x^2 + (\frac{8}{9} - a)x + \frac{25}{9} = 0$

(جواب پایین)

۳- $x=1 \rightarrow 4y = n+3 \Rightarrow y = \frac{n+3}{4}$, $\quad x=1 \rightarrow y = \frac{2+m}{4} \Rightarrow 2+m = n+3$

$4y = nx + 3x \Rightarrow y = \frac{nx+3x}{4x}$

$y' = \frac{(2x+m)(nx+3) - (x^2+mx+1)n}{(nx+3)^2} \Rightarrow \frac{4(2+m) - (2+m)}{16} = \frac{3}{4}$ ②

$\Rightarrow 12 = 6 + 3m \Rightarrow m = 2 \Rightarrow 4 = n+3 \Rightarrow n = 1 \quad \Rightarrow m+n = 1+2 = 3$ ✓

۴- $(2g-f)'(x) = 2g'(x) - f'(x) \quad 3 \times \frac{2}{2+\sin x} - \frac{(3-\sin x)(9+\sin^2 x + 3\sin x)}{(3-\sin x)(2+\sin x)} = \frac{6 - 9 - \sin^2 x - 3\sin x}{2+\sin x} = \frac{-4 - \sin^2 x - 2\sin x}{2+\sin x}$ ②

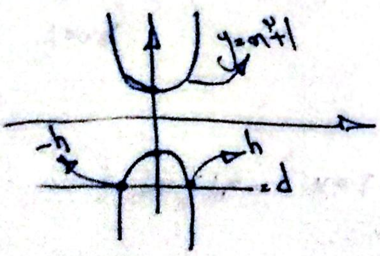
$\Rightarrow \frac{-\sin x(\sin x + 2)}{2+\sin x} \Rightarrow -\sin'(x) = -\cos x = -\cos\frac{5\pi}{3} = -\frac{1}{2}$ ✓

۵- $fog'(\sqrt[5]{2}) = g'(\sqrt[5]{2})\, f'(g(\sqrt[5]{2})) \Rightarrow g'(x) = \frac{0 - 10x^4}{(2x^5)^2} = \frac{-10x^4}{4x^{10}}$ ②

$g(x) = \frac{1}{2x^5}$

$fog(x) = \frac{-1}{\sqrt[5]{\frac{1}{2x^5}}} = \frac{-1}{\frac{1}{x}} = -x \Rightarrow fog'(x) = -1$ ✓

۶- $f'(x) = xg'(x) + g(x) \Rightarrow g(0) = f'(0) \Rightarrow f'(x) = 2\left(\frac{\sin x - 1}{\sin x + 1}\right) \times \frac{\cos x(1+\sin x) - \cos x(-1+\sin x)}{(1+\sin x)^2} = -2 \times 2 = -4$ ② ✓

۷- 

$y = -x^2 - 1 \Rightarrow y' = -2x \Rightarrow -2h \times 2h = -1 \Rightarrow 4h^2 = 1 \Rightarrow h = \pm\frac{1}{2}$

$\Rightarrow x = \frac{1}{2} \rightarrow y = -\frac{1}{4} - 1 = -\frac{5}{4}$ ✓ ②

$$f(x) = 2\sqrt{x}\,(4x^2+3) = 8x^2\sqrt{x} + 6\sqrt{x} \longrightarrow f'(x) = 20x\sqrt{x} + \frac{3}{\sqrt{x}} = \frac{20x^2+3}{\sqrt{x}}$$

−8

$$f'(x) = 8x \times 2\sqrt{x} - \frac{2}{2\sqrt{x}}(4x^2+3)$$

$$y = ax$$

$$\Rightarrow a = 16x\sqrt{x} - \frac{4x^2+3}{\sqrt{x}} = \frac{16x^2 - 4x^2 + 3}{\sqrt{x}} = \frac{12x^2+3}{\sqrt{x}}$$

$$2\sqrt{x}(4x^2+3) = 8x^2\sqrt{x} + 6\sqrt{x} = ax \Rightarrow \frac{8x^2+6}{\sqrt{x}} = a \Rightarrow 12x^2+3 = 8x^2+6$$

1

$$\Rightarrow 4x^2 - 3 = 0 \quad x = \pm \frac{3}{4} \quad x \neq -\frac{3}{4} \Rightarrow a = 16 \times \frac{3}{4}\sqrt{\frac{3}{4}} - \frac{12}{\sqrt{\frac{3}{4}}} = \frac{12 \times \frac{9}{4} + 3}{\sqrt{\frac{3}{4}}} = 30\sqrt{\frac{3}{4}}$$

$$y - 2\sqrt{\alpha}(4\alpha^2+3) = \frac{20\alpha^2+3}{\sqrt{\alpha}}(x-\alpha) \xrightarrow{(0,0)} -2\sqrt{\alpha}(4\alpha^2+3) = \frac{20\alpha^2+3}{\sqrt{\alpha}}(-\alpha)$$

$$\rightarrow 2(4\alpha^2+3) = 20\alpha^2 + 3 \rightarrow 12\alpha^2 = 3 \rightarrow \alpha^2 = \frac{1}{4}$$

$$m = \frac{20\left(\frac{1}{4}\right)+3}{\sqrt{\frac{1}{2}}} = 8\sqrt{2}$$

$$y = ax$$

$$f'(x) = \frac{\frac{1}{2\sqrt{x}}(-2x^2+x+1) - (-4x+1)(\sqrt{x})}{(-2x^2+x+1)^2} = a$$

−9

$$f(x) = ax \Rightarrow ax = \frac{\sqrt{x}}{-2x^2+x+1} = \frac{\sqrt{x}}{(x-1)\left(x+\frac{1}{2}\right)(-2)}$$

$$\Rightarrow a\sqrt{x} = \frac{1}{-2(x-1)\left(x+\frac{1}{2}\right)} \Rightarrow \frac{\sqrt{x}}{-2(x-1)\left(x+\frac{1}{2}\right)} = \frac{-x\sqrt{x} + \frac{1}{2}x + \frac{1}{2\sqrt{x}} - (-4x\sqrt{x}+\sqrt{x})}{-2x^2+x+1}$$

2

$$\Rightarrow \frac{-2x^2+x+1}{2\sqrt{x}} - \frac{(-8x^2+2)}{2\sqrt{x}} = \sqrt{x} \Rightarrow 6x^2+x-1 = 2x \Rightarrow 6x^2-x-1 = 0 \Rightarrow 6\left(x-\frac{3}{6}\right)\left(x+\frac{2}{6}\right) = 0$$

$$x = \frac{1}{2}$$

$$\Rightarrow f\left(\frac{1}{2}\right) = \frac{\sqrt{\frac{1}{2}}}{-\frac{1}{2}+\frac{1}{2}+1} = \sqrt{\frac{1}{2}}$$ ✓

0

−10

سوال ۲

$m = \frac{2-1}{2-(-1)} = \frac{1}{3}$

$f(x) = \sqrt{ax-1} \xrightarrow[\text{خط مماس}]{\text{شیب}} f'(x) = \frac{a}{2\sqrt{ax-1}} \to \frac{a}{2\sqrt{ax-1}} = \frac{1}{3} \to 3a = 2\sqrt{ax-1}$ (I)

معادله خط گذرنده از ۲ نقطه $= y = \frac{1}{3}x + \frac{4}{3} \to 3y = x+4 \to x+4 = 3\sqrt{ax-1}$ (II)

$\xrightarrow{\text{I و II}} x+4 = (\frac{3a}{2})3 = \frac{9a}{2} \to x = 4.5a - 4$

$\xrightarrow{\text{II}} 4.5a - 4 + 4 = 3\sqrt{a(4.5a-4)-1} \to 9a^2 - 16a - 4 = 0 \to \begin{cases} a = 2 \\ a = -\frac{2}{9} \end{cases}$

$f(5) = \sqrt{2(5)-1} = \sqrt{9} = 3$

سوال ۱۰

$(fog(\frac{\sqrt{5}}{2}))' = g'(\frac{\sqrt{5}}{2})^- \times f'(g(\frac{\sqrt{5}}{2})^-)$

$g(x) = (x^2-1)^{-\frac{1}{2}} \to g'(x) = -\frac{1}{2}(x^2-1)^{-\frac{3}{2}} \times 2x \to g'(\frac{\sqrt{5}}{2}^-) = \frac{1}{\sqrt{(\frac{5}{4})^- - 1}} = \frac{1}{\sqrt{(\frac{1}{4})^-}} = \frac{1}{(\frac{1}{2})^-} = 2^+$

$f'(2^+) = ((2x)^3)' = (8x^3)' = 24x^2 = 24 \times 4$

$\to g'(\frac{\sqrt{5}}{2}^-) \times f'(g(\frac{\sqrt{5}}{2})^-) = -4\sqrt{5} \times 4 \times 24 \to \frac{24 \times 4 \times (-4\sqrt{5})}{-48\sqrt{5}} = 8$