نام و نام خانوادگی ......[illegible]............ پاسخنامه تشریحی تکلیف شماره ۲۴... کلاس ...یازدهم تجربی A........

۱

$f'(3) = m_{خط} \Rightarrow (0,1),\ (3,5) \Rightarrow \frac{5-1}{3-0} = \frac{4}{3} \Rightarrow f'(3) = \frac{4}{3}$

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$m_{خط} = \frac{2-1}{2-(-1)} = \frac{1}{3} \xrightarrow[(-1,+1)]{معادله خط} y-1 = \frac{1}{3}(x+1) \Rightarrow y = \frac{1}{3}x + \frac{4}{3}$

نقطه مماس $\xrightarrow[\Delta=0]{معادله تلاقی} \sqrt{ax-1} = \frac{1}{3}x + \frac{4}{3} \Rightarrow 9ax - 9 = x^2 + 8x + 16$

$x^2 + (8-9a)x + 25 = 0 \Rightarrow \Delta = (8-9a)^2 - 4(25) = 0$

$8-9a = 10 \Rightarrow a = -\frac{2}{9} \xrightarrow{f(5)} \sqrt{-\frac{10}{9}-1}$: غ ق ق

$8-9a = -10 \Rightarrow a = 2 \xrightarrow{f(5)} \sqrt{2\times5-1} = \sqrt{9} = \boxed{3}$

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$y' = \frac{(2x+m)(x+3) - (x^2+mx+1)(1)}{(x+3)^2} = \frac{x^2+6x+3m-1}{(x+3)^2} \xrightarrow{x=1} \frac{6+3m}{16} = \frac{3}{4} \Rightarrow 6+3m = 12$ $\boxed{m=2}$

$m_{خط} \Rightarrow 4y = 3x+n \Rightarrow y = \frac{3}{4}x + \frac{n}{4} \Rightarrow m_{خط} = \frac{3}{4}$ (شیب)

$y = \frac{x^2+2x+1}{x+3} \xrightarrow{x=1} y = \frac{1+2+1}{4} = 1 \Rightarrow (1,1) \rightarrow$ درهر دو صدق می‌کند

$\Rightarrow 4y - 3x = n \xrightarrow{(1,1)} 4-3 = \boxed{1=n}$ $\Rightarrow m+n = 2+1 = \boxed{3}$

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$f(x) = \frac{27-\sin^3 x}{9-\sin^2 x} = \frac{\cancel{(3-\sin x)}(9+\sin^2 x+3\sin x)}{\cancel{(3-\sin x)}(3+\sin x)} = \frac{\sin^2 x + 3\sin x + 9}{3+\sin x}$

$3g'\left(\frac{5\pi}{3}\right) - f'\left(\frac{5\pi}{3}\right) = (3g-f)'\left(\frac{5\pi}{3}\right)$

$(3g-f)(x) = \frac{\cancel{9} - \sin^2 x - 3\sin x - \cancel{9}}{3+\sin x} = -\sin x \Rightarrow (3g-f)'(x) = -\cos x$

$(3g-f)'\left(\frac{5\pi}{3}\right) = -\cos\frac{5\pi}{3} = \boxed{-\frac{1}{2}}$

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$g'(\sqrt[5]{3})\, f'(g(\sqrt[5]{3})) = (fog)'(\sqrt[5]{3})$

$fog(x)_{x>0} = \frac{-1}{\sqrt[5]{\frac{1}{2x^5} + \frac{1}{2x^5}}} = \frac{-1}{\frac{1}{x}} = -x \Rightarrow (fog)'(x) = -1$

$(fog)'(\sqrt[5]{3}) = \boxed{-1}$

$g(x)_{x>0} = \frac{1}{2x^5}$

$f(x) = x\,g(x) + 1 \Rightarrow g(x) = \dfrac{f(x)-1}{x}$

$$\lim_{x\to 0} g(x) = \lim_{x\to 0} \frac{f(x)-1}{x} = \lim_{x\to 0} \frac{\left(\frac{-1+\sin x}{1+\sin x}\right)^2 - 1}{x} \xrightarrow{\text{هم‌ارزی}} \frac{\left(\frac{x-1}{x+1}\right)^2-1}{x} = \frac{\frac{\cancel{x^2}-2x+\cancel{1}-\cancel{x^2}-2x-\cancel{1}}{x^2+2x+1}}{x}$$ ۶

$$= \frac{-4\cancel{x}}{\cancel{x}(x^2+2x+1)} = \frac{-4}{(x+1)^2} \overset{x=0}{=} \boxed{-4}$$

قرینه نسبت به محور x → y قرینه $\Rightarrow -y = x^2+1 \Rightarrow y = -x^2-1$

$\left.\begin{matrix} y=-x^2-1 \\ y=K \end{matrix}\right\} \rightarrow K = -x^2-1 \Rightarrow x^2 = -K-1$

$x = \pm\sqrt{-K-1}$

$y' = -2x \longrightarrow x = +\sqrt{-K-1} \Rightarrow y' = -2\sqrt{-K-1}$ ۷

$\hookrightarrow x = -\sqrt{-K-1} \Rightarrow y' = +2\sqrt{-K-1}$

$\Big\} \xrightarrow[\text{حاصل‌ضرب} = -1]{\text{عمود}} -4(-K-1) = -1$

$(-1 > K)$

$-K-1 = \frac{1}{4}$

$-K = \frac{5}{4} \rightarrow K = -\frac{5}{4}$ ✓

$\Rightarrow y = -\frac{5}{4} \xrightarrow{\text{فاصله از مبدأ}} \boxed{\frac{5}{4}}$

خط d و $f(x)$ هر دو از مبدأ می‌گذرند $\Rightarrow (0,0)$: نقطه تقاطع $\Rightarrow f'(0) =$ شیب خط

$f'(x) = \dfrac{\cancel{2}}{\cancel{2}\sqrt{x}}(4x^2+3) + 2\sqrt{x}(8x) \Rightarrow f'(0) = \underbrace{\frac{1}{0}(3)}_{\text{ت.ن}} + 0 = \text{ت.ن} \Rightarrow$ موازی محور yها

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$\Rightarrow$ خط d $\Rightarrow x=0 \rightarrow$ شیب = تعریف نشده

خط d $\Rightarrow y = Kx$

$\left.\begin{matrix} y = Kx \\ f(x) = \dfrac{\sqrt{x}}{-2x^2+x+1} \end{matrix}\right\} \Rightarrow Kx = \dfrac{\sqrt{x}}{-2x^2+x+1} \Rightarrow K = \dfrac{1}{\sqrt{x}(-2x^2+x+1)}$

$f(\frac{1}{2}) = \dfrac{\frac{\sqrt{2}}{2}}{\cancel{-\frac{1}{2}+\frac{1}{2}}+1} = \boxed{\frac{\sqrt{2}}{2}}$

شیب‌ها $\rightarrow y' = K$ ۹

$$f'(x) = \frac{\frac{1}{2\sqrt{x}}(-2x^2+x+1) - \sqrt{x}(-4x+1)}{(-2x^2+x+1)^2} = \frac{-2x^2+x+1+8x^2-2x}{2\sqrt{x}(-2x^2+x+1)^2} = \frac{6x^2-x+1}{2\sqrt{x}(-2x^2+x+1)^2} \Big\} \Rightarrow$$

$$K = \frac{6x^2-x+1}{2\sqrt{x}(-2x^2+x+1)^{\cancel{2}}} = \frac{1}{\sqrt{x}\,\cancel{(-2x^2+x+1)}} \Rightarrow 6x^2-x+1 = -4x^2+2x+2$$

$10x^2-3x-1 = 0 \Rightarrow x = \dfrac{3 \pm \sqrt{9+40}}{20}$ → $\frac{1}{2}$, $-\frac{1}{4}$ غ ق ق

$fog(x) = \left(\dfrac{1}{\sqrt{x^2-1}}\left[\dfrac{1}{\sqrt{x^2-1}}\right]\right)^2 \Rightarrow fog(x) = \left(\dfrac{2}{\sqrt{x^2-1}}\right)^2 = \dfrac{4}{x^2-1} = 4(x^2-1)^{-1}$ ۱۰

$x < \frac{\sqrt{5}}{2} \Rightarrow x^2 < \frac{5}{4} \Rightarrow x^2-1 < \frac{1}{4} \Rightarrow \sqrt{x^2-1} < \frac{1}{2} \Rightarrow \dfrac{1}{\sqrt{x^2-1}} > 2 \Rightarrow \left[\dfrac{1}{\sqrt{x^2-1}}\right] = 2$

$\Rightarrow (fog)'(x) = -4(x^2-1)^{-2}(2x) \xrightarrow{x=\frac{\sqrt{5}}{2}} -4\left(\frac{1}{4}\right)^{-2}\left(2\times\frac{\sqrt{5}}{2}\right) = -4\times16\times\sqrt{5} = -64\sqrt{5}$

$\Rightarrow \dfrac{-64\sqrt{5}}{-48\sqrt{5}} = \boxed{\frac{4}{3}}$