نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ..... کلاس .....

۱

$g(x)=ax+b$ مماس $\xrightarrow{x=3} 3a+1=5 \to a=\frac{4}{3}$ $g'(x)=f'(x)=\frac{4}{3} \to$

$f'(3)=\frac{4}{3}$

(۲)

۲

$a=\frac{1-2}{-1-2}=\frac{1}{3} \Rightarrow d: y=\frac{1}{3}x+\frac{4}{3}$ $d=f(x) \Rightarrow \frac{1}{3}x+\frac{4}{3}=\sqrt{ax-1}$

$\to ax-1=\frac{x^2}{9}+\frac{8x}{9}+\frac{16}{9} \to x^2+(8-9a)x+25=0$ تک ریشه $\Rightarrow \Delta=0$

$\to (8-9a)^2=100 \to 8-9a=\pm 10 \begin{cases} \to a=-\frac{2}{9} \to f(a)=\sqrt{\frac{-18}{9}} \text{ ق ق} \\ \to a=2 \to f(a)=\sqrt{9}=3 \end{cases}$

(۲)

۳

$y_1=y_2 \xrightarrow{x=1} \frac{3+n}{4}=\frac{2+m}{4} \to m=n+1$ $y=\frac{3}{4}x+\frac{7}{4}$

$y_1'=y_2' \to \frac{(2x+m)(x+3)-(x^2+mx+1)}{(x+3)^2}=\frac{3}{4} \Rightarrow 2m=4 \to m=2$

$\to n=m-1 \to n=1$ $m+n=3$

(۲)

۴

$3g'(x)-f'(x)=(3g(x)-f(x))'$ $f(x)=\frac{(2-\sin x)(9+\sin^2 x+3\sin x)}{(2-\sin x)(3+\sin x)}$

$=\frac{\sin^2 x+3\sin x+9}{3\sin x}$ $3g(x)-f(x)=\frac{-\sin^2 x-3\sin x}{3+\sin x}=-\sin x \to$

$(3g(x)-f(x))'=-\cos x \xrightarrow{x=\frac{5\pi}{3}} -\cos\left(\frac{5\pi}{3}\right)=-\frac{1}{2}$

(۲)

۵

$g'(x)f'(g(x))=(fog(x))'$ $\sqrt[3]{x^3}>0 \to |x|=x \to fog(x)=-\frac{1}{\sqrt[3]{x^3}}=-x$

$\to (fog(x))'=-1$

(۲)

۶

$f(x)=xg(x)+1 \to g(x)=\frac{f(x)-1}{x}$ $\lim_{x\to 0} g(x)=\lim_{x\to 0}\frac{f(x)-1}{x}=\frac{0}{0}$

hop $\to \lim_{x\to 0} f'(x)=\lim_{x\to 0} 2\left(\frac{\sin x-1}{\sin x+1}\right)\left(\frac{2\cos x}{(1+\sin x)^2}\right)=-2\times 2\times 1=-4$ ✗

(۲)

۸

$y=-x^2-1$ شیب خط مماس $y'=-2x$

$-2x\times 2x=-1 \to 4x^2=1 \to x=\pm 1/2$ $\begin{cases}(-1/2,-5/4)\\(1/2,-5/4)\end{cases}\Rightarrow y=-\frac{5}{4}$

فاصله $=|d|=|-\frac{5}{4}|=\frac{5}{4}$ ✓ (۲)

۸

$d: ax+b \to d=f(x)$ و $d'=f'(x) \Rightarrow dx=f(x)x \to$

$ax=\frac{x}{\sqrt{x}}(4x^2+3x)+12x^2\sqrt{x} \to 2\sqrt{x}(4x^2+3x)+12x^2\sqrt{x} \to$

$4x^3-12x^2+3 \to 12x^2=3 \to x=\pm 1/2$ و $f:[0,+\infty) \Rightarrow x=+1/2$ (۲)

$f(1/2)=2\sqrt{1/2}\left(4\times\frac{1}{8}+3\right)=4\sqrt{2}$ $y=mx \to x=\frac{y}{m} \to m=\frac{4\sqrt{2}}{1/2}=8\sqrt{2}$ ✓

۹

$x=A \to Aa=\frac{\sqrt{A}}{-2A^2+A+1} \to a=\left(\frac{1}{2\sqrt{A}}(-2A^2+A+1)-\sqrt{A}(-4A+1)\right)\times$ (۱٫۵)

$\frac{1}{(-2A^2+A+1)^2}=\frac{-2A^2+A+1+8A^2-2A}{2\sqrt{A}}\times\frac{1}{(-2A^2+A)^2} \to \frac{\sqrt{A}}{A}=\frac{6A^2-A+1}{2\sqrt{A}(-2A^2+A+1)}$

$\to 10A^2-2A^2-A=0 \to \begin{cases}A_1=0\\A_2=-1/2\\A_3=1/2\end{cases}$ ✓ و غیق $f(x)=\frac{\sqrt{\frac{1}{2}}}{-2(\frac{1}{2})^2+\frac{1}{2}+1}=\frac{\sqrt{2}}{2}$

۱۰

$fog(x)=y'=g'(x)\times f'(g(x)) \xrightarrow{x=\sqrt{5}} y'(\sqrt{5})=g'(\frac{\sqrt{5}}{2})\times f'(g\frac{\sqrt{5}}{2})$

$g(\frac{\sqrt{10}}{2})=4^+ \to y'(\frac{\sqrt{5}}{2})=g'(\frac{\sqrt{5}}{2})\times f'_+(4)$ $f(x)=(x^2)^2 \to f'(x)=4x^3$

$f'_+(4)=96$ $g'(x)=-\frac{1}{4}\times 2x(x^2-1)^{-\frac{3}{4}} \to g'(\sqrt{5})=-5\sqrt{5}$ (۴)

$y'(\sqrt{5})=-5\sqrt{5}\times 96=-480\sqrt{5}$ $\frac{8x-8\sqrt{5}}{-48\sqrt{5}}=1$ ✓