آرینا حسینی تکلیف ریاضی شماره ۲۴ گروه دوازدهم دفتر A

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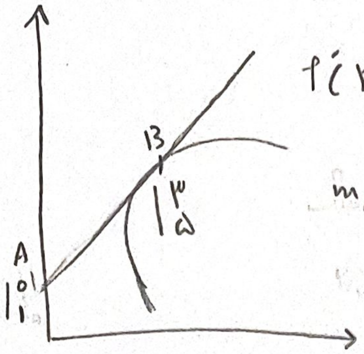

$(3,5)$

$f'(3)=?$

$m_{AB} = \dfrac{5-1}{3-0} = \dfrac{4}{3}$

2

2

$f(x) = \sqrt{ax-1} \longrightarrow f'(x) = \dfrac{a}{2\sqrt{ax-1}} \longrightarrow f(5) = \sqrt{2\times 5-1} = 3$

$(3,2)$, $(-1,1)$ از ۲ نقطه گذشته $\longrightarrow$ شیب $= \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{2-1}{2-(-1)} = \dfrac{1}{3} \to y-1 = \dfrac{1}{3}(x+1) \to y = \dfrac{1}{3}x + \dfrac{4}{3} \to 3y - x = 4$

$f(5)=?\quad \sqrt{ax-1} = \dfrac{x+4}{3} \to ax-1 = \left(\dfrac{x+4}{3}\right)^2 \to 9ax - 9 = x^2 + 8x + 16$

$\longrightarrow x^2 + (8-9a)x + 25 = 0 \longrightarrow \Delta = 0 \longrightarrow (8-9a)^2 - 4\times 25 = 0$

$\to 8-9a = 10 \to -2 = 9a \to \cancel{a = \dfrac{-2}{9}}$

$\to 8-9a = -10 \to 18 = 9a \to a = 2$

2

3

$y = \dfrac{x^2+mx+1}{x+3} \to y' = \dfrac{(2x+m)(x+3) - (x^2+mx+1)}{(x+3)^2} \longrightarrow \dfrac{2x^2+(m+6)x+3m-x^2-mx-1}{(x+3)^2}$

$4y-3x=n$, شیب $= \dfrac{3}{4}$

$y' = \dfrac{x^2+6x+3m-1}{(x+3)^2} = \dfrac{3}{4}$, $(x+3)^2 = x^2+6x+9$

$\to 4x^2 + 24x + 12m - 4 = 3x^2 + 18x + 27$

$\to x^2 + 6x + 12m - 31 = 0 \xrightarrow{x=1} 7 + 12m - 31 = 0 \to 12m = 24 \to m = 2$

$y = \dfrac{x^2+2x+1}{x+3} \xrightarrow{x=1} \dfrac{1+2+1}{1+3} = \dfrac{4}{4} = 1 \longrightarrow$ نقطه تماس $(1,1)$

$\longrightarrow 4(1) - 3(1) = n \longrightarrow n = 1 \longrightarrow m+n = 2+1 = 3$

2

4

$f(x) = \dfrac{27 - \sin^3 x}{9 - \sin^2 x} = \dfrac{\cancel{(3-\sin x)}(9 + 3\sin x + \sin^2 x)}{\cancel{(3-\sin x)}(\sin x + 3)} = \dfrac{9 + 3\sin x + \sin^2 x}{\sin x + 3}$

$g(x) = \dfrac{3}{3+\sin x}$

$3g'(x) - f'(x)$

$(3g(x) - f(x))' = \dfrac{\cancel{3\times 3} - \cancel{9} - 3\sin x - \sin^2 x}{3+\sin x} = \dfrac{-\sin x\cancel{(\sin x + 3)}}{\cancel{3+\sin x}} = (-\sin x)' = -\cos x$

$-\cos\left(\dfrac{5\pi}{3}\right) = -\dfrac{1}{2}$

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$$g'(\sqrt[5]{3})\, f'(g(\sqrt[5]{3})) = (f\circ g(x))' = (x') = \cancel{\boxed{1}} \quad -1$$

$$g(x) = \frac{1}{x^5 + |x^5|}$$

$$f(x) = \frac{-1}{\sqrt[5]{x+|x|}} \xrightarrow[x^5>0]{} f\circ g(x) = \frac{1}{\sqrt[5]{\frac{1}{|x^5|+x^5} + \frac{1}{|x^5+x^5|}}} = \frac{1}{\sqrt[5]{\frac{1}{2x^5}+\frac{1}{2x^5}}} = \frac{1}{\sqrt[5]{\frac{\cancel{2}}{\cancel{2}x^5}}} = \cancel{x} \quad -x$$

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$$y = x^2+1 \xrightarrow{\text{قرینه نسبت به } x} y = -x^2-1 \to y' = -2x$$

$$\text{خط } d \to y = k \to -x^2-1 = k \to x^2 = -k-1 \begin{cases} \to x = \sqrt{-k-1} \\ \to x = -\sqrt{-k-1} \end{cases} \to \text{نقاط برخورد}$$

$$(-2x)(2x) = -1 \to -4x^2 = -1 \to x^2 = \frac{1}{4} \to y = -\frac{1}{4} - 1 = \frac{-5}{4} \to y = -\frac{5}{4}$$

$\boxed{\text{فاصله تا مبدا مختصات } \frac{5}{4}}$

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سؤال ۲

$$f(x) = x\,g(x)+1 \to g(x) = \frac{f(x)-1}{x} \to \lim_{x\to 0} g(x) = \frac{0}{0} \xrightarrow{hop} \frac{+f'(0)}{0} = f'(0)$$

$$f(x) = \left(\frac{-1+\sin x}{1+\sin x}\right)^2 \to f'(x) = 2\left(\frac{\cos x\,(1+\sin x) - \cos x\,(-1+\sin x)}{(1+\sin x)^2}\right) \times \left(\frac{-1+\sin x}{1+\sin x}\right)$$

$$\to f'(0) = 2 \times \left(\frac{2}{1}\right) \times (-1) = -4$$

سوال ۸

$f(x) = 2\sqrt{x}\,(4x^2+3) = 8x^2\sqrt{x} + 6\sqrt{x} \longrightarrow f'(x) = 20x\sqrt{x} + \frac{3}{\sqrt{x}} = \frac{20x^2+3}{\sqrt{x}}$

$y - 2\sqrt{\alpha}\,(4\alpha^2+3) = \frac{20\alpha^2+3}{\sqrt{\alpha}}(x-\alpha) \xrightarrow{(0,0)} -2\sqrt{\alpha}\,(4\alpha^2+3) = \frac{20\alpha^2+3}{\sqrt{\alpha}}(-\alpha)$

$\longrightarrow 2(4\alpha^2+3) = 20\alpha^2+3 \longrightarrow 12\alpha^2 = 3 \longrightarrow \alpha^2 = \frac{1}{4}$

$m = \frac{20\left(\frac{1}{4}\right)+3}{\sqrt{\frac{1}{2}}} = 8\sqrt{2}$

سوال ۹

خط $\rightarrow y = ax$ $\qquad A(\alpha, a\alpha)$

$f(\alpha) = \frac{\sqrt{\alpha}}{-2\alpha^2+\alpha+1} = a\alpha \longrightarrow a\sqrt{\alpha}\,(-2\alpha^2+\alpha+1) = 1 \rightarrow -2a\alpha^{\frac{5}{2}} + a\alpha^{\frac{1}{2}} + a\alpha^{\frac{3}{2}} = 1$

$\xrightarrow{\text{مشتق}} -5a\alpha^{\frac{3}{2}} + \frac{3}{2}a\alpha^{\frac{1}{2}} + \frac{1}{2}a\alpha^{-\frac{1}{2}} = 0 \xrightarrow[\times 2\sqrt{\alpha}]{\div a} -10\alpha^2+3\alpha+1 = 0 \longrightarrow \begin{cases} \alpha = \frac{-1}{5} \\ \alpha = \frac{1}{2} \end{cases}$

$f(\alpha) = \frac{\sqrt{\frac{1}{2}}}{-2\left(\frac{1}{2}\right)^2+\frac{1}{2}+1} = \frac{\sqrt{2}}{2}$

سوال ۱۰

$\left(fog\left(\frac{\sqrt{5}}{2}\right)\right)' = g'\left(\frac{\sqrt{5}}{2}\right)^- \times f'\left(g\left(\frac{\sqrt{5}}{2}\right)^-\right)$

$g(x) = (x^2-1)^{-\frac{1}{2}} \rightarrow g'(x) = \frac{-1}{2}(x^2-1)^{-\frac{3}{2}} \times 2x \longrightarrow g'\left(\frac{\sqrt{5}}{2}^-\right) = \frac{1}{\sqrt{\left(\frac{5}{4}\right)^- - 1}} = \frac{1}{\sqrt{\left(\frac{1}{4}\right)^-}} = \frac{1}{\left(\frac{1}{2}\right)^-} = 2^+$

$f'(2^+) = \left((2x)^3\right)' = (8x^3)' = 24x^2 = 24 \times 4$

$\longrightarrow g'\left(\frac{\sqrt{5}}{2}^-\right) \times f'\left(g\left(\frac{\sqrt{5}}{2}\right)^-\right) = -4\sqrt{5} \times 4 \times 24 \longrightarrow \frac{24 \times 4 \times (-4\sqrt{5})}{-48\sqrt{5}} = 8$