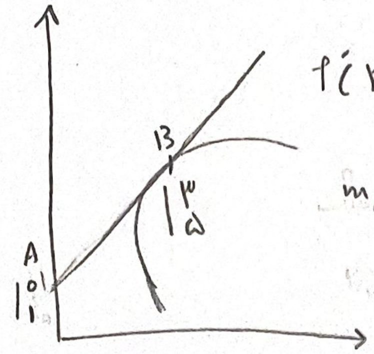
سرا ریناحسینیه تلخیص ریاضی شماره ۲۴ گروه دوازدهم دفتر A

۱

(۳،۵)



$f'(3) = ?$

$m_{AB} = \frac{5-1}{3-0} = \frac{4}{3}$

۲

$f(x) = \sqrt{ax-1} \longrightarrow f'(x) = \frac{a}{2\sqrt{ax-1}} \longrightarrow f(5) = \sqrt{2\times5-1} = 3$

(۲،۲) از ۲ نقطه بگذرد (۱،۱-) $\longrightarrow$ شیب $= \frac{y_2-y_1}{x_2-x_1} = \frac{2-1}{2-(-1)} = \frac{1}{3} \rightarrow y-1 = \frac{1}{3}(x+1) \rightarrow y = \frac{1}{3}x + \frac{4}{3} \rightarrow 3y - x = 4$

$f(5) = ?$ $\sqrt{ax-1} = \frac{x+4}{3} \rightarrow ax-1 = \left(\frac{x+4}{3}\right)^2 \rightarrow 9ax - 9 = x^2 + 8x + 16$

$\longrightarrow x^2 + (8-9a)x + 25 = 0 \longrightarrow \Delta = 0 \longrightarrow (8-9a)^2 - 4\times25 = 0$

$\rightarrow 8-9a = 10 \rightarrow -2 = 9a \rightarrow a = \frac{-2}{9}$ ✗

$\rightarrow 8-9a = -10 \rightarrow 18 = 9a \rightarrow a = 2$

۳

$y = \frac{x^2+mx+1}{x+3} \rightarrow y' = \frac{(2x+m)(x+3) - (x^2+mx+1)}{(x+3)^2} \longrightarrow \frac{2x^2+(m+6)x+3m-x^2-mx-1}{(x+3)^2}$

$4y - 3x = n$ ، شیب $= \frac{3}{4}$

$y' = \frac{x^2+6x+3m-1}{(x+3)^2} = \frac{3}{4}$ ، $(x+3)^2 = x^2+6x+9$

$\rightarrow 4x^2 + 24x + 12m - 4 = 3x^2 + 18x + 27$

$\rightarrow x^2 + 6x + 12m - 31 = 0 \xrightarrow{x=1} 7 + 12m - 31 = 0 \rightarrow 12m = 24 \rightarrow m = 2$

$y = \frac{x^2+2x+1}{x+3} \xrightarrow{x=1} \frac{1+2+1}{1+3} = \frac{4}{4} = 1 \longrightarrow$ نقطه تماس (۱،۱)

$\longrightarrow 4(1) - 3(1) = n \longrightarrow n = 1 \longrightarrow m+n = 2+1 = 3$

۴

$f(x) = \frac{27 - \sin^3 x}{9 - \sin^2 x} = \frac{(3-\sin x)(9+3\sin x+\sin^2 x)}{(3-\sin x)(\sin x+3)} = \frac{9+3\sin x+\sin^2 x}{\sin x+3}$

$g(x) = \frac{3}{3+\sin x}$

$3g'(x) - f'(x)$ ، $(3g(x) - f(x))' = \frac{3\times3 - 9 - 3\sin x - \sin^2 x}{3+\sin x} = \frac{-\sin x(\sin x+3)}{3+\sin x} = (-\sin x)' = -\cos x$

$-\cos\left(\frac{5\pi}{3}\right) = -\frac{1}{2}$

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$g'(\sqrt[3]{۳})\, f'(g(\sqrt[3]{۳}))\, (fog(x))' = (x')$ = ۱

$$g(x) = \frac{1}{x^3 + |x^3|}$$

$$f(x) = \frac{-1}{\sqrt[3]{x+|x|}} \xrightarrow[x^3>0]{} fog(x) = \sqrt[3]{\frac{1}{\frac{1}{|x^3|+x^3} + \frac{1}{|x^3|+x^3}}} = \sqrt[3]{\frac{1}{\frac{1}{2x^3} + \frac{1}{2x^3}}} = \sqrt[3]{\frac{1}{\frac{2}{2x^3}}} = x$$

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$y = x^2 + 1 \xrightarrow{\text{قرینه نسبت به } x} y = -x^2 - 1 \rightarrow y' = -2x$

خط $y = k \longrightarrow -x^2 - 1 = k \rightarrow x^2 = -k-1 \begin{cases} \rightarrow x = \sqrt{-k-1} \\ \rightarrow x = -\sqrt{-k-1} \end{cases}$ → نقاط برخورد

فاصله نقاط $\frac{5}{4}$

$(-2x)(2x) = -1 \longrightarrow -4x^2 = -1 \longrightarrow x^2 = \frac{1}{4} \longrightarrow y = -\frac{1}{4} - 1 = -\frac{5}{4} \rightarrow y = -\frac{5}{4}$