هلیا تاجی - ۱۲ دفتر A ۱۳٫۵

①

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-1}{3-0} = \frac{4}{3} \to y = \frac{4}{3}x + b$$

$f'(3) = m$ شیب مماس $\boxed{f'(3) = \frac{4}{3}}$ ✓

۲

---

②

$$m = \frac{2-1}{2+1} = \frac{1}{3} \to y = \frac{1}{3}x + \frac{4}{3} \xrightarrow{\text{تلاقی}} \frac{1}{3}x + \frac{4}{3} = \sqrt{ax-1}$$

$$\xrightarrow{\text{توان ۲}} x^2 + (8-9a)x + 25 = 0 \xrightarrow{\Delta = 0} (8-9a)^2 = 100 \to a = 2 ,\ a = -\frac{2}{9}$$

$$\xrightarrow{a=2 ✓} \sqrt{2x-1} \longrightarrow f(5) = \sqrt{9} = 3$$ ✓

۲

---

③

$$\frac{3}{4}x + n = \frac{x^2 + mx + 1}{x+3} \xrightarrow{x=1} \frac{3}{4} + n = \frac{2+m}{4} \qquad 3 + 4n = 2 + m$$

۱٫۵

$$y = \frac{3}{4}x + \frac{n}{4} \to \frac{3}{4} + \frac{n}{4} = \frac{2+m}{4} \to m - n = 1 \longrightarrow n = 1$$

$$\xrightarrow{f' = g'} \frac{3}{4} = \frac{(2x+m)(x+3) - (x^2+mx+1)}{(x+3)^2} \Rightarrow \frac{3}{4} = \frac{2x^2 + 6x + \cancel{2mx} + 3m - x^2 - \cancel{mx} - 1}{x^2 + 9 + 6x}$$

$$m + n = 2 + 1 = 3$$

$$\frac{3}{4} = \frac{x^2 + 6x + 3m - 1}{(x+3)^2} \xrightarrow{x=1} \frac{3}{\cancel{4}} = \frac{6 + 3m}{\cancel{16}\ 4} \qquad 3m = 6 \quad m = 2 \quad n = \frac{1}{4} \quad m + n = \frac{9}{4}$$

---

④

$$3g'\left(\frac{5\pi}{3}\right) - f'\left(\frac{5\pi}{3}\right) \to (3g - f)'_{\left(\frac{5\pi}{3}\right)} \to \frac{9}{3 + \sin x} - \frac{27 - \sin^3 x}{9 - \sin^2 x}$$

$$\to \frac{\cancel{9} - \cancel{9} - \sin^2 x - 3\sin x}{3 + \sin x} = \frac{-\sin x(3 + \cancel{\sin x})}{\cancel{\sin x + 3}} = -\sin x \qquad \frac{(3 - \sin x)(9 + \sin^2 x + 3\sin x)}{(3 - \sin x)(3 + \sin x)}$$

$$(-\sin x)' = -\cos x \longrightarrow -\cos\frac{5\pi}{3} = -\frac{1}{2}$$ ✓

۲

---

⑤

$$g(x) = \frac{1}{x^5 + |x^5|} \xrightarrow{x > 0} \frac{1}{2x^5} \to 2x^{-5} \qquad \Big|\ g'(\sqrt[5]{3})\, f'(g\,\sqrt[5]{3})$$

$$f(x) = \frac{-1}{\sqrt[5]{x + |x|}} \xrightarrow{x > 0} \frac{-1}{x^{\frac{2}{5}}} \qquad -x^{-\frac{2}{5}} \qquad \Big| \to fog\ \sqrt[5]{3}$$

۱

$$g'(x) \times f'(g(x)) = (fog)'(x)$$

$$fog(x) = -1 \times 2x^{-\cancel{5} \times -\frac{2}{\cancel{5}}} \longrightarrow -4x^2 \xrightarrow{\text{مشتق}} -8x \xrightarrow{x = \sqrt[5]{3}} -8\sqrt[5]{3}$$

$$x > 0 \to g(x) = \frac{1}{2x^5} ,\quad x > 0 \to f(x) = \frac{-1}{\sqrt[5]{2x}} \xrightarrow{x > 0} fog(x) = \frac{-1}{\sqrt[5]{2\left(\frac{1}{2x^5}\right)}}$$

$$\to fog(x) = -x \to (fog)'(x) = -1 \to (fog)'(\sqrt[5]{3}) = -1$$

۸

$f(x)=g(x) \longrightarrow ۲\sqrt{x}\,(۴x^۲+۳)=mx \xrightarrow[\text{ساده‌شده}]{} ۸x^۲+۶=m\sqrt{x}$

$f'(x)=g'(x) \longrightarrow ۱۶x=\frac{m}{۲\sqrt{x}} \longrightarrow m=۳۲x\sqrt{x}$

$۸x^۲+۶=۳۲x^۲ \longrightarrow x=\frac{۱}{۲} \xrightarrow{\text{جایگذاری}} m=۳۲x\sqrt{x}=۸\sqrt{۲}$ ✓

۲

۹

$f(x)=\left(\frac{-۱+\sin x}{۱+\sin x}\right)^۲ \quad f(x)=xg(x)+۱ \xrightarrow{\text{مشتق}} f'(x)=g(x)+xg'(x)$

$f'(0)=g(0) \longrightarrow f'(x)=۲\left(\frac{-۱+\sin 0}{۱+\sin 0}\right)\left(\frac{۲\cos 0+۲\sin 0\cos 0}{(۱+\sin 0)^۲}\right)$

$f'(0)=۲(-۱)(۲) \quad f'(0)=-۴$ ✓ $\rightarrow g(0)=-۴$

۲

۹

معادله → $y=mx$ ① $f(x)=g(x)$ $\quad mx=\frac{\sqrt{x}}{-۲x^۲+x+۱}$

② $f'(x)=g'(x) \sim \frac{\frac{۱}{۲\sqrt{x}}(-۲x^۲+x+۱)-(\sqrt{x})(-۴x+۱)}{(-۲x^۲+x+۱)^۲}=m \longrightarrow \frac{۱}{mx}=\frac{۲}{mx}+۴x\sqrt{x}-\sqrt{x}$

$\frac{۲}{mx} \qquad \left(\frac{\sqrt{x}}{mx}\right)^۲$

۱

$\frac{۱}{mx}=\sqrt{x}-۴x\sqrt{x} \qquad \cancel{m}x\cancel{\sqrt{x}}=۴\cancel{m}x^۲\cancel{\sqrt{x}} \qquad ۴x^۲-x=0 \begin{cases} x=0 \\ x=\frac{۱}{۴} \checkmark \end{cases}$

$x=\frac{۱}{۴} \longrightarrow y=\frac{\frac{۱}{۲}}{-\frac{۱}{۸}+\frac{۱}{۴}+۱}=\frac{\frac{۱}{۲}}{\frac{-۱+۲+۸}{۸}}=\frac{\frac{۱}{۲}}{\frac{۹}{۸}}=\frac{۴}{۹}$

خط → $y=ax \qquad A(\alpha, a\alpha)$

$f(\alpha)=\frac{\sqrt{\alpha}}{-۲\alpha^۲+\alpha+۱}=a\alpha \rightarrow a\sqrt{\alpha}(-۲\alpha^۲+\alpha+۱)=۱ \rightarrow -۲a\alpha^{\frac{۵}{۲}}+a\alpha^{\frac{۳}{۲}}+a\alpha^{\frac{۱}{۲}}=۱$

$\xrightarrow{\text{مشتق}} -۵a\alpha^{\frac{۳}{۲}}+\frac{۳}{۲}a\alpha^{\frac{۱}{۲}}+\frac{۱}{۲}a\alpha^{-\frac{۱}{۲}}=۰ \xrightarrow[\times ۲\sqrt{\alpha}]{\div a} -۱۰\alpha^۲+۳\alpha+۱=۰ \longrightarrow \begin{cases} \alpha=\frac{-۱}{۵} \\ \alpha=\frac{۱}{۲} \end{cases}$

$f(\alpha)=\frac{\sqrt{\frac{۱}{۲}}}{-۲(\frac{۱}{۲})^۲+\frac{۱}{۲}+۱}=\frac{\sqrt{۲}}{۲}$

سؤال ۷

$y = x^2 + 1 \xrightarrow[\text{محور } x]{\text{قرینه نسبت به}} y_1 = -(x^2+1) = -x^2 - 1 \xrightarrow{\text{مشتق}} y' = -2x$

خط d، سهمی y_1 را در نقطهٔ A و B قطع کند چون خط d موازی محور xها است ← نقاط A و B ← عرض یکسان و طول قرینه دارند.

$A(\alpha, \beta)$, $B(-\alpha, \bar{\beta}) \rightsquigarrow A(\frac{1}{2}, \beta)$, $B(-\frac{1}{2}, \beta)$

$mL_1 = y_1'(-\alpha) = -2(-\alpha) = 2\alpha$

$mL_2 = y_1'(\alpha) = -2\alpha$

عمود اند $L_1, L_2 \longrightarrow mL_1 \times mL_2 = -1 \longrightarrow -2\alpha \cdot 2\alpha = -1 \longrightarrow 4\alpha^2 = 1 \longrightarrow \alpha = \pm\frac{1}{2}$

فاصله خط از مبدا مختصات $\rightarrow |\beta| \longrightarrow \beta = y_1(\frac{-1}{2}) = -(-\frac{1}{2})^2 - 1 = -\frac{1}{4} - 1 = \frac{-5}{4} = -1{,}25 \longrightarrow |\beta| = 1{,}25$

سؤال ۱۰

$(f \circ g(\frac{\sqrt{5}}{2}))' = g'(\frac{\sqrt{5}}{2})^- \times f'(g(\frac{\sqrt{5}}{2})^-)$

$g(x) = (x^2-1)^{-\frac{1}{2}} \rightarrow g'(x) = \frac{-1}{2}(x^2-1)^{-\frac{3}{2}} \times 2x \rightarrow g'(\frac{\sqrt{5}}{2}^-) = \frac{1}{\sqrt{(\frac{5}{4})^- - 1}} = \frac{1}{\sqrt{(\frac{1}{4})^-}} = \frac{1}{(\frac{1}{2})^-} = 2^+$

$f'(2^+) = ((2x)^3)' = (8x^3)' = 24x^2 = 24 \times 4$

$\longrightarrow g'(\frac{\sqrt{5}}{2}^-) \times f'(g(\frac{\sqrt{5}}{2})^-) = -4\sqrt{5} \times 4 \times 24 \longrightarrow \frac{24 \times 4 \times (-4\sqrt{5})}{-48\sqrt{5}} = 8$