هیبا تاکی - ۱۲ دفتر A

(۱)

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{5-1}{3-0}=\frac{4}{3}\to y=\frac{4}{3}x+b$$

$f'(3)=m$ شیب مماس $\boxed{f'(3)=\frac{4}{3}}$

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(۲)

$$m=\frac{2-1}{2+1}=\frac{1}{3}\to y=\frac{1}{3}x+\frac{4}{3}\xrightarrow{\text{تلاقی}}\frac{1}{3}x+\frac{4}{3}=\sqrt{ax-1}$$

$$\xrightarrow{\text{توان ۲}} x^2+(8-9a)x+25=0\xrightarrow{\Delta=0}(8-9a)^2=100 \quad a=2,\ a=-\frac{2}{9}$$

$$\xrightarrow{a=2\checkmark}\sqrt{2x-1}\longrightarrow f(5)=\sqrt{9}=3$$

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(۳)

$$\frac{3}{4}x+n=\frac{x^2+mx+1}{x+3}\xrightarrow{x=1}\frac{3}{4}+n=\frac{2+m}{4}\qquad 3+4n=2+m$$

$$\xrightarrow{f'=g'}\frac{3}{4}=\frac{(2x+m)(x+3)-(x^2+mx+1)}{(x+3)^2}\Rightarrow\frac{3}{4}=\frac{2x^2+6x+\cancel{mx}+3m-x^2-\cancel{mx}-1}{x^2+9+6x}$$

$$\frac{3}{4}=\frac{x^2+6x+3m-1}{(x+3)^2}\xrightarrow{x=1}\frac{3}{\cancel{4}}=\frac{6+3m}{\cancel{16}\,4}\qquad 3m=6\quad m=2\quad n=\frac{1}{4}\quad m+n=\frac{9}{4}$$

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(۴)

$$3g'\left(\frac{5\pi}{3}\right)-f'\left(\frac{5\pi}{3}\right)\to(3g-f)'_{\left(\frac{5\pi}{3}\right)}\to\frac{9}{3+\sin x}-\frac{27-\sin^3x}{9-\sin^2x}$$

$$\frac{(\cancel{3-\sin x})(9+\sin^2x+3\sin x)}{(\cancel{3-\sin x})(3+\sin x)}$$

$$\to\frac{\cancel{9}-\cancel{9}-\sin^2x-3\sin x}{3+\sin x}=\frac{-\sin x(\cancel{3+\sin x})}{\cancel{\sin x+3}}=-\sin x$$

$$(-\sin x)'=-\cos x\longrightarrow-\cos\frac{5\pi}{3}=-\frac{1}{2}$$

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(۵)

$$g(x)=\frac{1}{x^5+|x^5|}\xrightarrow{x>0}\frac{1}{2x^5}\to 2x^{-5}$$

$g'(\sqrt[5]{3})\ f'(g\sqrt[5]{3})$

$-x^{-\frac{2}{5}}$ $\to fog\ \sqrt[5]{3}$

$$f(x)=\frac{-1}{\sqrt[5]{x+|x|}}\xrightarrow{x>0}\frac{-1}{x^{\frac{2}{5}}}$$

$$fog(x)=-1\times2x^{-\cancel{5}\times-\frac{2}{\cancel{5}}}\to-4x^2\xrightarrow[\text{مشتق}]{}-8x\xrightarrow{x=\sqrt[5]{3}}-8\sqrt[5]{3}$$

(8)

$f(x)=g(x) \longrightarrow 2\sqrt{x}\,(4x^2+3)=mx \xrightarrow{\text{ساده شده}} 8x^2+6=m\sqrt{x}$

$f'(x)=g'(x) \longrightarrow 16x=\dfrac{m}{2\sqrt{x}} \longrightarrow m=32\sqrt{x}$

$8x^2+6=32x^2 \longrightarrow \boxed{x=\dfrac{1}{2}} \xrightarrow{\text{جایگذاری}} m=32x\sqrt{x}=\boxed{8\sqrt{2}}$

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(4)

$f(x)=\left(\dfrac{-1+\sin x}{1+\sin x}\right)^2 \quad f(x)=xg(x)+1 \xrightarrow{\text{مشتق}} f'(x)=g(x)+xg'(x)$

$f'(0)=g(0) \longrightarrow f'(\underset{0}{x})=2\left(\dfrac{-1+\sin 0}{1+\sin 0}\right)\left(\dfrac{2\cos 0+2\sin 0\cos 0}{(1+\sin 0)^2}\right)$

$f'(0)=2(-1)(2) \quad f'(0)=-4 \longrightarrow \boxed{g(0)=-4}$

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(9)

مماس $\rightarrow y=mx$ ① $f(x)=g(x) \quad mx=\dfrac{\sqrt{x}}{-2x^2+x+1}$

② $f'(x)=g'(x) \sim \dfrac{\underbrace{\frac{1}{2\sqrt{x}}(-2x^2+x+1)}_{\frac{2}{mx}}-(\sqrt{x})(-4x+1)}{\underbrace{(-2x^2+x+1)^2}_{\left(\frac{\sqrt{x}}{mx}\right)^2}}=m \longrightarrow \dfrac{1}{mx}=\dfrac{2}{mx}+4x\sqrt{x}-\sqrt{x}$

$\dfrac{1}{mx}=\sqrt{x}-4x\sqrt{x}$

$\cancel{m}x\cancel{\sqrt{x}}=4\cancel{m}x^2\cancel{\sqrt{x}} \quad 4x^2-x=0 \begin{cases} x=0 \\ x=\frac{1}{4} \checkmark \end{cases}$

$x=\dfrac{1}{4} \longrightarrow y=\dfrac{\frac{1}{2}}{-\frac{1}{8}+\frac{1}{4}+1}=\dfrac{\frac{1}{2}}{\frac{-1+2+8}{8}}\ \dfrac{\frac{1}{2}}{\frac{9}{8}}=\boxed{\dfrac{4}{9}}$