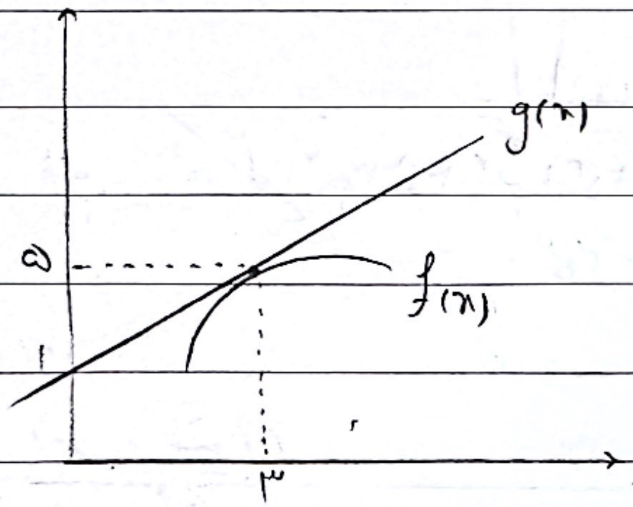
عربی

۱۷/۵

1)



$(x-x_0)(m) = y - y_0 \rightarrow 3m = 4 \rightarrow m = \frac{4}{3}$

$g(x) = ax + b \xrightarrow{b=1} g(x) = \frac{4}{3}x + 1$ ۲

f و g در $x = 3$ مماس هم اند $\rightarrow f'(3) = g'(3) = \frac{4}{3}$ ✓

صحیح شده

۲) $(-1,1)$, $(2,2)$ $\to m(x-x_0)=y-y_0 \to m=\frac{1}{3} \to g(x)=\frac{x}{3}+b \xrightarrow[b=\frac{4}{3}]{(-1,1)} g(x)=\frac{x+4}{3}$

مماس‌اند f, g : $f=g$, $\Delta=0 \to \frac{x+4}{3}=\sqrt{ax-1} \to ax-1=\frac{(x+4)^2}{9} \to$

$9ax-9=(x+4)^2 \to x^2+8x+16-9ax+9=0 \to x^2+x(8-9a)+25=0 \xrightarrow{\Delta=0}$

$(8-9a)^2+(-4)(1)(25)=0 \to (8-9a)^2=100 \nearrow 8-9a=10 \to a=\frac{-2}{9}$

$\searrow 8-9a=-10 \to a=2$

$f'(x)=\frac{a}{2\sqrt{ax-1}}$ مماس‌اند $f,g \to f'=g'$ در نقطه‌ی تماس

$a=2 \nearrow \frac{1}{\sqrt{2x-1}}=\frac{1}{3} \to \checkmark$

$a=-\frac{2}{9} \searrow \frac{\frac{2}{9}}{2\sqrt{\frac{-2x}{9}-1}}=\frac{-1}{9\sqrt{\frac{-2x}{9}-1}}=\frac{1}{3} \to \times$

(طرف دوم مثبت، طرف اول به دلیل منفی است)

۲

$\frac{1}{\sqrt{2x-1}}=\frac{1}{3} \xrightarrow{\text{توان } 2} 2x-1=9 \to x=5$

$f(5)=g(5)=\frac{9}{3}=3$ ✓

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۳) $y=\frac{x^2+mx+1}{x+3} \xrightarrow[\text{مماس}]{x=1 \text{ در}} 4y-3x=n \to m+n=?$

$f'(x)=\frac{(2x+m)(x+3)-(x^2+mx+1)}{(x+3)^2} \xrightarrow{x=1} f'(1)=\frac{(2+m)(4)-(2+m)}{16}=\frac{6+3m}{16}$

$g(x)=\frac{3x}{4}+\frac{n}{4} \to g'(x)=\frac{3}{4}$

$f'(x)=g'(x) \to \frac{6+3m}{16}=\frac{3}{4} \to 4=2+m \to m=2$

۲

s.a.m

$f(x) = \frac{x^2 + 2x + 1}{x + 3}$

$f(1) = g(1) \rightarrow \frac{4}{4} = 1 = \frac{3 + n}{4} \rightarrow n = 1$

$g(x) = \frac{3x + n}{4}$ ✓

$m + n = 2 + 1 = 3$

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۴) $f(x) = \frac{27 - \sin^3 x}{9 - \sin^2 x}$ $\rightarrow f(x) = \frac{(3 - \sin)(9 + \sin^2 + 3\sin)}{(3 - \sin)(3 + \sin)} = \sin + \frac{9}{3 + \sin}$

$g(x) = \frac{3}{3 + \sin x}$ $\qquad f'(x) = \cos x - \frac{9\cos}{(3 + \sin)^2} \xrightarrow{\frac{5\pi}{3}} \frac{1}{2} - \frac{4{,}5}{(3 - \frac{\sqrt{3}}{2})^2}$

$3g'(\frac{5\pi}{3}) - f'(\frac{5\pi}{3}) = ?$ $\qquad g'(x) = \frac{-3\cos}{(3 + \sin)^2} \xrightarrow{x = \frac{5\pi}{3}} \frac{-1{,}5}{(3 - \frac{\sqrt{3}}{2})^2}$

(۲)

$\hookrightarrow \frac{-4{,}5}{(3 - \frac{\sqrt{3}}{2})^2} - \frac{1}{2} + \frac{4{,}5}{(3 - \frac{\sqrt{3}}{2})^2} = -\frac{1}{2}$ ✓

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۵) $f(x) = \frac{-1}{\sqrt[5]{x + |x|}}$ $\xrightarrow{x > 0}$ $f(x) = \frac{1}{\sqrt[5]{2x}} = (2x)^{-\frac{1}{5}}$

$f o g = (2 \times \frac{1}{2} \times x^{-5})^{-\frac{1}{5}} = x$

(۱/۵)

$g(x) = \frac{1}{x^5 + |x^5|}$ $\xrightarrow{x > 0}$ $g(x) = \frac{1}{2x^5} = (2x^5)^{-1}$ $\qquad (f o g)'(x) = 1$

$f o g(x) = \frac{-1}{\sqrt[5]{2(\frac{1}{2x^5})}} \rightarrow f o g(x) = -x \rightarrow (f o g)'(x) = -1 \rightarrow (f o g)'(\sqrt[5]{3}) = -1$

$g'(\sqrt[5]{3})\, f'(g(\sqrt[5]{3})) = ?$ $\rightarrow (f o g)'(x) = ?$ 1

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۶) $f(x) = \left(\frac{\sin - 1}{1 + \sin}\right)^2$ $\rightarrow \left(\frac{(\sin - 1)(\sin + 1)}{(1 + \sin)^2}\right)^2 = \left(\frac{\sin^2 - 1}{(1 + \sin)^2}\right)^2 = \left(\frac{-\cos^2 x}{(1 + \sin)^2}\right)^2 =$

(۰/۵)

$f(x) = x g(x) + 1$ $\qquad f(x) = \left(\frac{\cos}{1 + \sin}\right)^4 = x g(x) + 1 \rightarrow g(x) = \frac{\tan^4 \frac{x}{4} - 1}{x}$

$\lim_{x \to 0} g(x) = ?$

$-\frac{1}{0}$ $\quad 0^+ \rightarrow -\infty$ $\quad 0^- \rightarrow +\infty$ $\quad$ حد ندارد

s.a.m

$f(x) = x g(x) + 1 \rightarrow g(x) = \frac{f(x) - 1}{x} \rightarrow \lim_{x \to 0} g(x) = \frac{0}{0} \xrightarrow{hop} \frac{f'(0)}{1} = f'(0)$

$f(x) = \left(\frac{-1 + \sin x}{1 + \sin x}\right)^2 \rightarrow f'(x) = 2\left(\frac{\cos x(1 + \sin x) - \cos x(-1 + \sin x)}{(1 + \sin x)^2}\right) \times \left(\frac{-1 + \sin x}{1 + \sin x}\right)$

$\rightarrow f'(0) = 2 \times \left(\frac{2}{1}\right) \times (-1) = -4$

۷) $y=x^2+1$ قرینه نسبت به محورها ... منقطع در نقطه‌ای از خط $y=d$ ... مماس‌های آن ... فاصله‌ی (۰,۰) ... دو نقطه متقاطع‌اند

قرینه نسبت به محور x ها: $-y=x^2+1 \to y=-x^2-1$, $-x^2-1=d \to -1+d=x^2 \to x=\pm\sqrt{d-1}$

$y'=2x \to y'=2\sqrt{d-1}$, $\to y'=-2\sqrt{d-1}$

$2\sqrt{d-1}=\frac{1}{2\sqrt{d-1}} \to 4(d-1)=1 \to d-1=\frac{1}{4}$

$d=\frac{5}{4}$

2

$y=\frac{5}{4}$ فاصله از (۰,۰) $=\frac{5}{4}$ ✓

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۸) $g(x)=kx \to$ خط d $\to g'(x)=k$

$f(x)=2\sqrt{x}(4x^2+3) \to f(x)=8x^{\frac{5}{2}}+6x^{\frac{1}{2}} \to f'(x)=20x^{\frac{3}{2}}+3x^{-\frac{1}{2}}$

$f(x)=g(x)$, $\Delta=0 \to kx=2\sqrt{x}(4x^2+3) \to k=\frac{8x^2+6}{\sqrt{x}}$

1/8

$g'(x)=f'(x) \to k=20x^{\frac{3}{2}}+3x^{-\frac{1}{2}}$

$20x\sqrt{x}+\frac{3}{\sqrt{x}}=\frac{8x^2+6}{\sqrt{x}} \xrightarrow{\times\sqrt{x}} 20x^2+3=8x^2+6$

$12x^2=3 \to x=\pm\frac{1}{2}$

$D_f: x\geq 0 \to \begin{cases} x=\frac{1}{2} ✓ \\ x=-\frac{1}{2} ✗ \end{cases}$

$g(\frac{1}{2})=f(\frac{1}{2}) \to \frac{9}{\sqrt{2}}=\frac{k}{2} \to \frac{18\sqrt{2}}{2}=9\sqrt{2}=k$

$\to y=9\sqrt{2}x \to$ شیب خط d $=9\sqrt{2}$

$y-2\sqrt{\alpha}(4\alpha^2+3)=\frac{20\alpha^2+3}{\sqrt{\alpha}}(x-\alpha) \xrightarrow{(0,0)} -2\sqrt{\alpha}(4\alpha^2+3)=\frac{20\alpha^2+3}{\sqrt{\alpha}}(-\alpha)$

$\to 2(4\alpha^2+3)=20\alpha^2+3 \to 12\alpha^2=3 \to \alpha^2=\frac{1}{4}$

$m=\frac{20(\frac{1}{4})+3}{\sqrt{\frac{1}{2}}}=8\sqrt{2}$

s.a.m

۹) $f(x) = \dfrac{\sqrt{x}}{-2x^2 + x + 1}$

خط d : $g(x) = kx$ → مماس در نقطه A → x چند ?

$A(a, f(a))$ → $m_d = \dfrac{f(a) - 0}{a - 0} = \dfrac{f(a)}{a}$

(۲)

$f'(a) = g'(a) = \dfrac{f(a)}{a} \to \dfrac{\left(\frac{1}{2\sqrt{x}}\right)(-2x^2+x+1) - (-4x+1)(\sqrt{x})}{(-2x^2+x+1)(-2x^2+x+1)} = \dfrac{\sqrt{x}}{\sqrt{x}\,(x)(-2x^2+x+1)}$

$10x^2 - 3x - 1 = 0 \to x^2 - 3x - 10 = 0 \to (x-5)(x+2) = 0$ $\begin{cases} x = \frac{5}{10} = \frac{1}{2} & ✓ \\ x = \frac{-2}{10} = \frac{-1}{5} & ✗ \end{cases}$

$f\left(\frac{1}{2}\right) = \dfrac{\frac{1}{\sqrt{2}}}{-\frac{1}{2} + \frac{1}{2} + 1} = \dfrac{\sqrt{2}}{2}$ ✓

با توجه به دامنه $x \geq 0$

۱۰) $f(x) = (x[x])^3 \to f'(x) = 3([x])(x[x])^2$

$g(x) = \dfrac{1}{\sqrt{x^2-1}} = (x^2-1)^{-1}$ $\quad g'(x) = -(x^2-1)^{-2} \times 2x \xrightarrow{x = \frac{\sqrt{5}}{2}} g'(x) = -4\sqrt{5}$

$x < \frac{\sqrt{5}}{2} \to x^2 < \frac{5}{4} \to x^2 - 1 < \frac{1}{4} \to -\frac{1}{2} < \sqrt{x^2-1} < \frac{1}{2}$

$\dfrac{(fog)'_-\left(\frac{\sqrt{5}}{2}\right)}{-48\sqrt{5}} = \to \begin{cases} g(x) > 2 \xrightarrow{g(x)>0} ✓ \to g\left(\frac{\sqrt{5}}{2}\right) = 2^+ \\ g(x) < -2 \xrightarrow{g(x)>0} ✗ \end{cases}$

(۲)

$f'(g(x)) = f'(2^+) = 3(2)(4)^2 = 96$

$\dfrac{(fog)'\left(\frac{\sqrt{5}}{2}\right)^-}{-48\sqrt{5}} = \dfrac{f'\left(g\left(\frac{\sqrt{5}}{2}\right)^-\right) \times g'\left(\frac{\sqrt{5}}{2}\right)}{-48\sqrt{5}} = \dfrac{96 \times -4\sqrt{5}}{-48\sqrt{5}} = 8$ ✓