نام و نام خانوادگی .... پاسخنامه تشریحی تکلیف شماره ۲۴ ... کلاس دوازدهم ... دبیر جبر A

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$m = \frac{5-1}{3-0} = \frac{4}{3} \Rightarrow f'(3) = \frac{4}{3}$

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$m = \frac{2-1}{2-(-1)} = \frac{1}{3} \rightarrow y-1 = \frac{1}{3}(x+1) \rightarrow y = \frac{1}{3}x + \frac{4}{3}$

$\xrightarrow[\text{مماس}]{\Delta = 0} \sqrt{ax-1} = \frac{1}{3}x + \frac{4}{3} \rightarrow 9ax + (-9) = x^2 + 8x + 16 \rightarrow x^2 + (8-9a)x + 25 = 0$

$\xrightarrow{\Delta = 0} (8-9a)^2 - 4(25) = 0 \begin{cases} 8-9a = 10 \rightarrow a = \frac{-2}{9} \rightarrow f(5) = \sqrt{\frac{-10}{9} - 1} \text{ غ ق ق} \\ 8-9a = -10 \rightarrow a = 2 \rightarrow f(5) = \sqrt{9} = 3 \end{cases}$

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$y' = \frac{(2x+m)(x+3) - (x^2+mx+1)(1)}{(x+3)^2} = \frac{x^2+6x+3m-1}{(x+3)^2} \xrightarrow{x=1} \frac{6+3m}{16} = \frac{3}{4} \rightarrow m = 2$

$4y = 3x + n \rightarrow y = \frac{3}{4}x + \frac{n}{4} \rightarrow m_{\text{خط}} = \frac{3}{4}$

$x = 1 \rightarrow y = \frac{4}{4} = 1$ (نقطه تماس در معادله خط صدق می‌کند) $\quad 4 = 3 + n \ (1,1) \rightarrow n = 1$

$m + n = 3$

(۲)

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$f(x) = \frac{(3-\sin x)(9+\sin^2 x+3\sin x)}{(3-\sin x)(3+\sin x)} = \frac{\sin^2 x + 3\sin x + 9}{3+\sin x}$

$3g'(\frac{5\pi}{3}) - f'(\frac{5\pi}{3}) = (3g-f)'(\frac{5\pi}{3})$

$(3g-f)(x) = \frac{9 - \sin^2 x - 3\sin x - 9}{3+\sin x} = -\sin x \rightarrow (3g-f)'(x) = -\cos x$

$\xrightarrow{x = \frac{5\pi}{3}} -\cos\frac{5\pi}{3} = ~~-\frac{\sqrt{3}}{2}~~$ $\quad -\frac{1}{2}$

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$g'(\sqrt[5]{3})\, f'(g(\sqrt[5]{3})) = (fog)'(\sqrt[5]{3})$

$fog(x) = \frac{-1}{\sqrt[5]{\frac{2}{x^{5}}}} \rightarrow \frac{-1}{\frac{1}{x}} = -x \rightarrow (fog)'(x) = -1$

$(fog)'(\sqrt[5]{3}) = -1$

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$f(x) = x\,g(x) + 1$

$\Downarrow$

$g(x) = \dfrac{f(x)-1}{x}$

$\Downarrow$

$\lim_{x\to 0} g(x) = \lim_{x\to 0} \dfrac{f(x)-1}{x} = \lim_{x\to 0} \dfrac{\left(\frac{-1+\sin x}{1+\sin x}\right)^2 - 1}{x} \xrightarrow{\text{هوپیتال}} \dfrac{-4x}{x(\dots)} \xrightarrow{x=0} \boxed{-4}$ ✓

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قرینه نسبت به محور x $\rightarrow -y = x^2+1 \rightarrow y = -x^2-1,\ y = k \gg k = -x^2-1 \rightarrow x = \pm\sqrt{-k-1}$

$y' = -2x \rightarrow y' = -2\sqrt{-k-1}$ ، $y' = +2\sqrt{-k-1}$ $\rightarrow$ عمود $= -1 \rightarrow -4(-k-1) = -1$

$-k = \dfrac{5}{4} \rightarrow k = \dfrac{-5}{4}$

$y = -\dfrac{5}{4} \xrightarrow{\text{عرض مبدا}} \boxed{+\dfrac{5}{4}}$ ✓

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مساحت ... $f(x)$ و d ... $\rightarrow (0,0)$ نقطه تماس $\rightarrow f'(0)$ شیب خط

$f'(x) = \dfrac{1}{\sqrt{x}}(4x^2+x) + 2\sqrt{x}(8x+1) \longrightarrow f'(0) = \dfrac{1}{0} \dots \Rightarrow$ تعریف نشده $\rightarrow$ خط مماس عمود است

$x=0 \longrightarrow$ ...

(جواب پایین صفر)

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خط $d \rightarrow y = ax \qquad A(\alpha, a\alpha)$

$f(\alpha) = \dfrac{\sqrt{\alpha}}{-2\alpha^2+\alpha+1} = a\alpha \rightarrow a\sqrt{\alpha}(-2\alpha^2+\alpha+1) = 1 \rightarrow -2a\alpha^{\frac{5}{2}} + a\alpha^{\frac{3}{2}} + a\alpha^{\frac{1}{2}} = 1$

$\xrightarrow{\text{مشتق}} -5a\alpha^{\frac{3}{2}} + \frac{3}{2}a\alpha^{\frac{1}{2}} + \frac{1}{2}a\alpha^{-\frac{1}{2}} = 0 \xrightarrow[\times 2\sqrt{\alpha}]{\div a} -10\alpha^2 + 3\alpha + 1 = 0 \longrightarrow \begin{cases} \alpha = \frac{-1}{5} \\ \alpha = \frac{1}{2} \end{cases}$

$f(\alpha) = \dfrac{\sqrt{\frac{1}{2}}}{-2(\frac{1}{2})^2 + \frac{1}{2} + 1} = \dfrac{\sqrt{2}}{2}$

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$fog(x) = \left(\dfrac{1}{\sqrt{x^2-1}}\left[\dfrac{1}{\sqrt{x^2-1}}\right]\right)^2 \rightarrow fog(x) = \dfrac{4}{x^2-1} = 4(x^2-1)^{-1}$

$x < \dfrac{\sqrt{5}}{2} \rightarrow x^2 < \dfrac{5}{4} \rightarrow x^2-1 < \dfrac{1}{4} \rightarrow \sqrt{x^2-1} < \dfrac{1}{2} \rightarrow \dfrac{1}{\sqrt{x^2-1}} > 2$

$(fog)'(x) = -4(x^2-1)^{-2}(2x) \xrightarrow{x = \frac{\sqrt{5}}{2}} -4 \times 16 \times \sqrt{5} = -64\sqrt{5}$

$\dfrac{-64\sqrt{5}}{-48\sqrt{5}} = \dfrac{4}{3}$

(جواب پایین صفر)

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سوال ۸)

$$f(x) = 2\sqrt{x}\,(4x^2+3) = 8x^2\sqrt{x} + 6\sqrt{x} \longrightarrow f'(x) = 20x\sqrt{x} + \frac{3}{\sqrt{x}} = \frac{20x^2+3}{\sqrt{x}}$$

$$y - 2\sqrt{\alpha}\,(4\alpha^2+3) = \frac{20\alpha^2+3}{\sqrt{\alpha}}(x-\alpha) \xrightarrow{(0,0)} -2\sqrt{\alpha}\,(4\alpha^2+3) = \frac{20\alpha^2+3}{\sqrt{\alpha}}(-\alpha)$$

$$\longrightarrow 2(4\alpha^2+3) = 20\alpha^2 + 3 \longrightarrow 12\alpha^2 = 3 \longrightarrow \alpha^2 = \frac{1}{4}$$

$$m = \frac{20\left(\frac{1}{4}\right)+3}{\sqrt{\frac{1}{2}}} = 8\sqrt{2}$$

سوال ۱۰)

$$\left(fog\left(\frac{\sqrt{5}}{2}\right)\right)' = g'\left(\frac{\sqrt{5}}{2}\right)^- \times f'\left(g\left(\frac{\sqrt{5}}{2}\right)^-\right)$$

$$g(x) = (x^2-1)^{-\frac{1}{2}} \longrightarrow g'(x) = -\frac{1}{2}(x^2-1)^{-\frac{3}{2}} \times 2x \longrightarrow g'\left(\frac{\sqrt{5}}{2}^-\right) = \frac{1}{\sqrt{\left(\frac{5}{4}\right)^- - 1}} = \frac{1}{\sqrt{\left(\frac{1}{4}\right)^-}} = \frac{1}{\left(\frac{1}{2}\right)^-} = 2^+$$

$$f'(2^+) = \left((2x)^3\right)' = (8x^3)' = 24x^2 = 24 \times 4$$

$$\longrightarrow g'\left(\frac{\sqrt{5}}{2}^-\right) \times f'\left(g\left(\frac{\sqrt{5}}{2}\right)^-\right) = -4\sqrt{5} \times 4 \times 24 \longrightarrow \frac{24 \times 4 \times (-4\sqrt{5})}{-48\sqrt{5}} = 8$$