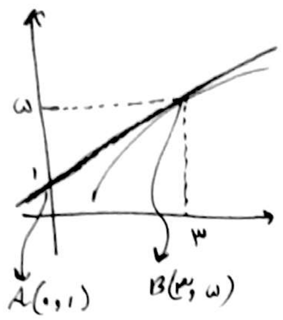
نام و نام خانوادگی ...[illegible]... **۱۹٫۲۵** پاسخنامه تشریحی تکلیف شماره ...۲۴... کلاس ...دوازدهم دختر A...

۱ (۱٫۷۵)



$f'(3) = \frac{5-1}{3-0} = \frac{4}{3}$

۲ (۲)

$(-1,1)\ /\ (2,2) \rightarrow m = \frac{2-1}{2-(-1)} = \frac{1}{3} \rightarrow y = \frac{1}{3}(x+1)+1 \Rightarrow y = \frac{1}{3}x + \frac{4}{3}$ → معادله خط مماس

$\frac{1}{3}x + \frac{4}{3} = \sqrt{ax-1} \rightarrow x^2 + (8-9a)x + 25 = 0 \quad \Delta = 0 \rightarrow (8-9a)^2 - 100 = 0$

$\Rightarrow a = 2, -\frac{2}{9} \xrightarrow{a=2} f(x) = \sqrt{2x-1} \rightarrow f(5) = \boxed{3}$

۳ (۲)

$4y - 3x = n \rightarrow$ شیب $= \frac{3}{4}$ ؛ $f(x) = \frac{x^2+mx+1}{x+3}$ ، $f'(1) = \frac{3}{4}$ $\Rightarrow f'(x) = \frac{x^2+6x+3m-1}{(x+3)^2} \xrightarrow{x=1} \frac{6+3m}{16} = \frac{3}{4}$

$\Rightarrow 2+m = 4 \Rightarrow \boxed{m=2}$

$f(1) = \frac{1+(2)(1)+1}{1+3} = \frac{4}{4} = 1 \Rightarrow 4y - 3 = n \Rightarrow y = \frac{n+3}{4} = 1 \Rightarrow n+3 = 4 \Rightarrow \boxed{n=1}$

$m+n = 2+1 = \boxed{3}$

۴ (۲)

$f(x) = \frac{(3-\sin x)(\sin^2 x + 9 + 3\sin x)}{(3-\sin x)(3+\sin x)} \Rightarrow f(x) = \frac{\sin^2 x + 3\sin x + 9}{3+\sin x}$ / $3g(x) = \frac{9}{3+\sin x}$

$3g'(\frac{5\pi}{3}) - f'(\frac{5\pi}{3}) = \left(3g(\frac{5\pi}{3}) - f(\frac{5\pi}{3})\right)' \rightarrow 3g(x) - f(x) = \frac{9 - \sin^2 x - 3\sin x - 9}{3+\sin x} = \frac{-\sin x(\sin x + 3)}{3+\sin x}$

$\Rightarrow \left(-\sin(\frac{5\pi}{3})\right)' = -\cos(\frac{5\pi}{3}) = \boxed{-\frac{1}{2}}$

۵ (۱٫۵)

$g'(\sqrt[5]{3})\, f'(g(\sqrt[5]{3})) = \left(f(g(\sqrt[5]{3}))\right)'$ ؛ $f(x) = \frac{1}{\sqrt[5]{x+|x|}} \xrightarrow{\text{دامنه} +} f(x) = \frac{1}{\sqrt[5]{2x}}$

$g(x) = \frac{1}{x^5 + |x^5|} \xrightarrow{\text{دامنه} +} g(x) = \frac{1}{2x^5}$ ؛ $fog(x) = \frac{-1}{\sqrt[5]{2(\frac{1}{2x^5})}}$

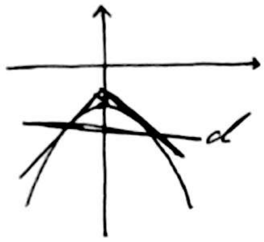
$\rightarrow fog = f(\frac{1}{2x^5}) = x \rightarrow (x)' = \boxed{1}$

$\rightarrow fog(x) = -x \rightarrow (fog)'(x) = -1 \rightarrow (fog)'(\sqrt[5]{3}) = -1$

$g(x) = \frac{f(x)-1}{x} \to \lim_{x\to 0} \frac{f(x)-1}{x} \xrightarrow{f(0)=1} \lim_{x\to 0} \frac{f(x)-f(0)}{x-0} = f'(0)$

$f'(x) = 2\left(\frac{-1+\sin x}{1+\sin x}\right)(\cos x)\left(\frac{2}{(\sin x+1)^2}\right) \xrightarrow{x=0} f'(0) = (2)(-1)(1)(2) = \boxed{-4}$ ✓ ۶

(۲)



$\to f(x) = -x^2 - 1$ (قرینه نسبت به محور x) / $g(x) = k$ (خط d) / $-x^2 - 1 = k \Rightarrow x^2 = -k-1$

$\Rightarrow$ طول‌ها $= \pm\sqrt{-k-1} \to f'(x) = -2x$ / $m_1 = -2\sqrt{-k-1}$, $m_2 = 2\sqrt{-k-1}$ (۲) ۷

$\xrightarrow{m_1 \cdot m_2 = -1} (-2\sqrt{-k-1})(2\sqrt{-k-1}) = -1 \Rightarrow -4(-k-1) = -1 \Rightarrow 4k+4 = -1 \Rightarrow k = -\frac{5}{4}$

$\Rightarrow g(a) = -\frac{5}{4} \longrightarrow$ عرض از مبدأ $= \frac{5}{4}$ ✓

$\begin{cases} f(x) = g(x) \Rightarrow 2\sqrt{x}(4x^2+3) = m\sqrt{x} \Rightarrow 8x^2+6 = m\sqrt{x} \\ f'(x) = g'(x) \ \text{-----------} \ x \end{cases} \to 16x = \frac{m}{2\sqrt{x}}$

(۲) ۸

$\Rightarrow m = 32x\sqrt{x} \Rightarrow 8x^2+6 = 32x^2 \xrightarrow{x>0} x = \frac{1}{2}$ $\xrightarrow{\text{جایگذاری}}$ $m = 32x\sqrt{x}$

$\Rightarrow \boxed{m = 8\sqrt{2}}$ ✓

خط $= mx$ / $f(x) = \frac{\sqrt{x}}{-2x^2+x+1}$

$\to$ نقطه‌ی تماس $\to x = a \Rightarrow m = \frac{f(a)}{a} = f'(a)$ (از مبدأ می‌گذرد) $\Rightarrow f'(x) = \frac{\frac{-2x^2+x+1}{2\sqrt{x}} - \sqrt{x}(-4x+1)}{(-2x^2+x+1)^2}$ $= \boxed{\frac{\sqrt{2}}{2}}$ (۲)

طول نقطه A _ عرض ۹

$f(x) = \frac{f(x)}{x} = \frac{1}{x(-2x^2+x+1)} \Rightarrow \frac{1}{\sqrt{x}(-2x^2+x+1)} = \frac{(-4x+1)\sqrt{x} - \frac{-2x^2+x+1}{2\sqrt{x}}}{(-2x^2+x+1)^2} \xrightarrow{\text{ساده}} \Rightarrow (2x-1)(5x+1) = 0$

$\Rightarrow x = \frac{1}{2}$ $(x \neq -\frac{1}{5})$

$x \to \left(\frac{\sqrt{5}}{2}\right)^- \Rightarrow \sqrt{x^2-1} \to \left(\frac{1}{2}\right)^- \Rightarrow g(x) \to 2^+$

$(f(g(x)))' \xrightarrow{\left(\frac{\sqrt{5}}{2}\right)^-} g'_-\left(\frac{\sqrt{5}}{2}\right) \cdot f'_+(2)$ / $g'(x) = \left((x^2-1)^{-\frac{1}{2}}\right)' = -\frac{1}{2}(x^2-1)^{-\frac{3}{2}}(2x)$

$\Rightarrow g'_-\left(\frac{\sqrt{5}}{2}\right) = -4\sqrt{5}$ / $x \to 2^+ \Rightarrow f(x) = ($[illegible]$)^3 \Rightarrow f'(x) = 24x^2 \Rightarrow f'_+(2) = 96$ ۱۰

$\to (-4\sqrt{5})(96) = 8(-48\sqrt{5}) \Rightarrow$ ۸ برابر ✓ (۲)