Day: Month: Year: ۱۸ Subject: مبینا رضایی

(۱

$A(۳,۵)$ $B(۰,۱)$ $\xrightarrow{\text{شیب}} m = \frac{۵-۱}{۳-۰} = \frac{۴}{۳}$

شیب خط برابر است با $f'(x)$ $\longleftarrow$ $f'(۳) = \frac{۴}{۳}$ ✓

(۲)

(۲

$(۲,۲)\ ,\ (-۱,۱) \longrightarrow m = \frac{۱}{۳} \longrightarrow y = \frac{۱}{۳}x + \frac{۴}{۳}$

$\frac{۱}{۳}x + \frac{۴}{۳} = \sqrt{ax-۱} \xrightarrow{\times ۳} x + ۴ = ۳\sqrt{ax-۱} \xrightarrow{\text{توان ۲}} x^۲ + ۸x + ۱۶ = ۹ax - ۹$

(۲)

$\rightarrow x^۲ + (۸ - ۹a)x + ۲۵ = ۰ \xrightarrow{\Delta = ۰} (۸-۹a)^۲ - ۴(۲۵) = ۰$

$f(۵) = \sqrt{-\frac{۱}{۹} - ۱}$ غ ق ق $\qquad f(۵) = \sqrt{۲\times۵-۱} = \boxed{۳}$ ✓

$a = \frac{-۲}{۹}$ یا $a = ۲$

(۳

$۴y - ۳x = n \longrightarrow m = \frac{۳}{۴}$

(۲)

مشتق تابع در $x=۱$ $= \frac{(۲x+m)(۴) - (۲+m)}{۱۶} = \frac{۳(۲+m)}{۱۶} = \frac{۳}{۴} \rightarrow \boxed{m=۲}$

$x=۱ \rightarrow y=۱ \xrightarrow{۴y-۳x=n} ۴-۳=۱ \rightarrow n=۱ \qquad m+n = \boxed{۳}$

(۴

$(۳g - f)'(\frac{۵\pi}{۳}) = \frac{۹}{۳+\sin x} - \frac{(۳-\sin x)(۹+۳\sin x+\sin^۲ x)}{(۳-\sin x)(۳+\sin x)}$

(۲)

$= \frac{۹-۹-۳\sin x-\sin^۲ x}{۳+\sin x} = \frac{-\sin(۳+\sin x)}{۳+\sin x} = (-\sin x)' = -\cos\frac{۵\pi}{۳} = \boxed{-\frac{۱}{۲}}$ ✓

(۵

$$g'(\sqrt[5]{3})\,f'(g(\sqrt[5]{3})) = f(g(\sqrt[5]{3}))'$$

۲

$$\lim_{x\to 0} f\circ g(x) = \frac{-1}{\sqrt[5]{\frac{1}{2x^5}+\frac{1}{2x^5}}} = \frac{-1}{\sqrt[5]{\frac{2}{2x^5}}} = \frac{-1}{\frac{1}{x}} = -x \xrightarrow{\text{مشتق}} -1$$

(۶

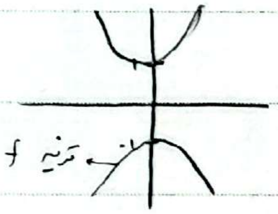
$$g(x) = \frac{f(x)-1}{x} \xrightarrow[\text{هم‌ارزی}]{\sin x \approx x} \frac{\left(\frac{1+x}{1-x}\right)^2 - 1}{x} = \frac{\frac{x^2+2x+1-x^2-2x-1}{x^2-2x+1}}{x}$$

۲

$$= \frac{-4x}{x(x^2+2x+1)} = \frac{-4}{(x+1)^2} \xrightarrow{x=0} \boxed{-4}$$

(۷

$-1 > k \leftarrow y=k$ خط $\leftarrow$



$$-y = x^2+1 \to x^2 = -k-1 \to x = \pm\sqrt{-k-1}$$

۲

$$y' = -2x \longrightarrow \begin{cases} x = +\sqrt{-k-1} \to y' = -2\sqrt{-k-1} \\ x = -\sqrt{-k-1} \to y' = +2\sqrt{-k-1} \end{cases} \xrightarrow{\text{ضرب}} -4(-k-1) = -1$$

$$k = -\frac{5}{4}$$

$k = \frac{5}{4} \leftarrow$ با ملاک مبدا $\leftarrow$

(۸ ۱

① $2\sqrt{x}\,(4x^2+3)' = ax \longrightarrow a = \frac{2\sqrt{x}\,(4x^2+3)}{x}$

② $\frac{2}{2\sqrt{x}} \times (4x^2+3) + 8x(2\sqrt{x}) = a \xrightarrow{\sqrt{x}} 4x^2+3+16x^2 = a\sqrt{x}$

$f(x) = 2\sqrt{x}(4x^2+3) = 8x^2\sqrt{x}+6\sqrt{x} \longrightarrow f'(x) = 20x\sqrt{x} + \frac{3}{\sqrt{x}} = \frac{20x^2+3}{\sqrt{x}}$

a جایگذاری $\longrightarrow 20x^2+3 = \frac{2x(4x^2+3)}{x} \longrightarrow 20x^2+3 = 8x^2+6 \to x^2 = 4$

① در x جایگذاری $\longrightarrow 2\sqrt{2} \times 19 = 2a \longrightarrow a = 19\sqrt{2}$ $\quad m = \frac{20(\frac{1}{4})+3}{\sqrt{\frac{1}{4}}} = 8\sqrt{4}$ $\quad x = \pm 2$

[illegible] $\quad \boxed{x = 2}$

$$y - 2\sqrt{\alpha}(4\alpha^2+3) = \frac{20\alpha^2+3}{\sqrt{\alpha}}(x-\alpha) \xrightarrow{(0,0)} -2\sqrt{\alpha}(4\alpha^2+3) = \frac{20\alpha^2+3}{\sqrt{\alpha}}(-\alpha)$$

$$\to 2(4\alpha^2+3) = 20\alpha^2+3 \to 12\alpha^2 = 3 \to \alpha^2 = \frac{1}{4}$$

$y = ax$

$f(x) = \dfrac{\sqrt{x}}{-2x^2+x+1}$ ① $ax = \dfrac{\sqrt{x}}{-2x^2+x+1} \longrightarrow a = \dfrac{1}{\sqrt{x}(-2x^2+x+1)}$ (9

② $a = \dfrac{\frac{1}{2\sqrt{x}}(-2x^2+x+1) - (-4x+1)\sqrt{x}}{(-2x^2+x+1)^2} \xrightarrow[\times 2\sqrt{x}]{\times 2\sqrt{x}} \dfrac{-2x^2+x+1+8x^2-2x}{2\sqrt{x}\,(-2x^2+x+1)^2}$

$= \dfrac{6x^2-x+1}{2\sqrt{x}(-2x^2+x+1)^2} = a \rightarrow \dfrac{6x^2-x+1}{2\sqrt{x}(-2x^2+x+1)^2} = \dfrac{1}{\sqrt{x}(-2x^2+x+1)}$

$6x^2 - x + 1 = -4x^2 + 2x + 2 \longrightarrow 10x^2 - 3x - 1 = 0$ $x = \frac{1}{2}$ ✓

$x = \frac{-1}{5}$ غ ق ق

(4)

$f(\frac{1}{2}) = \dfrac{\sqrt{2}}{2}$ ✓

$fog(x) = \left(\dfrac{1}{\sqrt{x^2-1}}\left[\dfrac{1}{\sqrt{x^2-1}}\right]\right)^2 \longrightarrow fog(x) = \left(\dfrac{2}{\sqrt{x^2-1}}\right)^2 = \dfrac{4}{x^2-1}$ (10

$x < \frac{\sqrt{5}}{2} \longrightarrow x^2 < \frac{5}{4} \longrightarrow x^2 - 1 < \frac{1}{4} \rightarrow \sqrt{x^2-1} < \frac{1}{2} \rightarrow \dfrac{1}{\sqrt{x^2-1}} > 2$ (1)

$\longrightarrow \left[\dfrac{1}{\sqrt{x^2-1}}\right] = 2$ $(fog)'(x) = \dfrac{-8x}{(x^2-1)^2} \xrightarrow{x = \frac{\sqrt{5}}{2}} -64\sqrt{5}$

$\dfrac{-64\sqrt{5}}{-48\sqrt{5}} = \dfrac{4}{3}$

$(fog(\frac{\sqrt{5}}{2}))' = g'(\frac{\sqrt{5}}{2})^- \times f'(g(\frac{\sqrt{5}}{2})^-)$

$g(x) = (x^2-1)^{-\frac{1}{2}} \rightarrow g'(x) = \frac{-1}{2}(x^2-1)^{-\frac{3}{2}} \times 2x \rightarrow g'(\frac{\sqrt{5}}{2}^-) = \dfrac{1}{\sqrt{(\frac{5}{4})^- - 1}} = \dfrac{1}{\sqrt{(\frac{1}{4})^-}} = \dfrac{1}{(\frac{1}{2})^-} = 2^+$

$f'(2^+) = ((2x)^3)' = (8x^3)' = 24x^2 = 24 \times 4$

$\rightarrow g'(\frac{\sqrt{5}}{2}^-) \times f'(g(\frac{\sqrt{5}}{2})^-) = -4\sqrt{5} \times 4 \times 24 \longrightarrow \dfrac{24 \times 4 \times (-4\sqrt{5})}{-48\sqrt{5}} = 8$