۱۷٫۲۵

۱)
$f'(3) = m \longrightarrow \begin{matrix}(0,1)\\(3,5)\end{matrix} \longrightarrow \frac{5-1}{3-0} = \frac{4}{3} \longrightarrow f'(3) = \frac{4}{3}$ ✓

(۲)

۲)
$m = \frac{2-1}{2+1} = \frac{1}{3} \longrightarrow y-1 = \frac{1}{3}(x+1) \longrightarrow y = \frac{1}{3}x + \frac{4}{3}$

خط مماس: معادله تلاقی، $\Delta = 0$: $\sqrt{ax-1} = \frac{1}{3}x + \frac{4}{3} \longrightarrow 9ax - 9 = x^2 + 8x + 16 \longrightarrow$

$x^2 + (8-9a)x + 25 = 0 \longrightarrow \Delta = (8-9a)^2 - 100 = 0 \longrightarrow 8 - 9a = 10 \longrightarrow a = -\frac{2}{9} \longrightarrow f(5) = \sqrt{-\frac{10}{9} - 1}$ غ ق ق

$\hookrightarrow 8 - 9a = -10 \longrightarrow a = 2 \longrightarrow f(5) = 3$ ✓

(۲)

۳)
$y' = \frac{(2x+m)(x+3) - (x^2+mx+1)}{(x+3)^2} = \frac{x^2+6x+3m-1}{(x+3)^2} \xrightarrow{x=1} \frac{6+3m}{16} = \frac{3}{4} \longrightarrow m = 2$

$4y = 3x + n \longrightarrow y = \frac{3}{4}x + \frac{n}{4} \longrightarrow$ شیب خط $= \frac{3}{4}$

$y = \frac{x^2+2x+1}{x+3} \xrightarrow{x=1} y = \frac{4}{4} = 1 \longrightarrow (1,1) \longrightarrow$ در معادله صدق می‌کند

$4y - 3x = n \longrightarrow 4 - 3 = 1 = n \longrightarrow m + n = 3$ ✓

(۲)

۴)
$f(x) = \frac{27 - \sin^3 x}{9 - \sin^2 x} = \frac{(3 - \sin x)(9 + \sin^2 x + 3\sin x)}{(3 - \sin x)(3 + \sin x)} = \frac{\sin^2 x + 3\sin x + 9}{3 + \sin x}$

$3g'(\frac{5\pi}{3}) - f'(\frac{5\pi}{3}) = (3g - f)'(\frac{5\pi}{3})$ $\quad -\cos(\frac{5\pi}{3}) = \frac{-1}{2}$

$(3g - f)(x) = \frac{9 - \sin^2 x - 3\sin x - 9}{3 + \sin x} = -\sin x \longrightarrow (3g-f)'(x) = -\cos x \longrightarrow (3g-f)'(\frac{5\pi}{3}) = -\frac{\sqrt{3}}{2}$

(۱٫۷۵)

۵)
$g'(\sqrt[5]{3})\, f'(g(\sqrt[5]{3})) = (f\circ g)'(\sqrt[5]{3})$ ، $\underset{x>0}{g(x)} = \frac{1}{2x^5}$

$\underset{x>0}{f\circ g(x)} = \frac{-1}{\sqrt[5]{\frac{1}{2x^5} + \frac{1}{2x^5}}} = \frac{-1}{\frac{1}{x}} = -x \longrightarrow (f\circ g)'(x) = -1 \longrightarrow (f\circ g)'(\sqrt[5]{3}) = -1$ ✓

(۲)

۶)
$f(x) = xg(x) + 1 \longrightarrow g(x) = \frac{f(x) - 1}{x}$

$\lim_{x\to 0} g(x) = \lim_{x\to 0} \frac{f(x)-1}{x} = \lim_{x\to 0} \frac{\left(\frac{-1+\sin x}{1+\sin x}\right)^2 - 1}{x} \longrightarrow \frac{\left(\frac{x-1}{x+1}\right)^2 - 1}{x} = \frac{-4x}{x(x^2+2x+1)} = \frac{-4}{(x+1)^2} = -4$ ✓

(۲)

۷)
$-y = x^2 + 1 \longrightarrow y = -x^2 - 1$ ، $y = k$ $\longrightarrow k = -x^2 - 1 \longrightarrow x^2 = -k - 1 \longrightarrow x = \pm\sqrt{-k-1}$

$y' = -2x \longrightarrow x = \sqrt{-k-1} \longrightarrow y' = -2\sqrt{-k-1}$

$\hookrightarrow x = -\sqrt{-k-1} \longrightarrow y' = 2\sqrt{-k-1}$

$\longrightarrow -4(-k-1) = -1 \longrightarrow -k-1 = \frac{1}{4} \longrightarrow k = -\frac{5}{4}$

$y = -\frac{5}{4} \longrightarrow$ فاصله از مبدا $= \frac{5}{4}$ ✓

(۲)

۸) ۰/۵

نقطه تقاطع $f(x)$ و d $= (0,0) \to f'(0) =$ شیب خط

$$f'(x) = \frac{2}{2\sqrt{x}}(4x^2+3) + 2\sqrt{x}\,(8x) \to f'(0) = \frac{1}{0}(3)+0 = \text{ت.ن}$$ → موازی محور y

→ شیب خط: تعریف نشده

(جواب پایین صفحه)

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۹)

خط: $y = kx$

$$f(x) = \frac{\sqrt{x}}{-2x^2+x+1} \Big\} \to kx = \frac{\sqrt{x}}{-2x^2+x+1} \to k = \frac{1}{\sqrt{x}(-2x^2+x+1)}$$

$y' = k$

$$f'(x) = \frac{\frac{1}{2\sqrt{x}}(-2x^2+x+1) - \sqrt{x}(-4x+1)}{(-2x^2+x+1)^2} = \frac{6x^2-x+1}{2\sqrt{x}(-2x+x+1)^2} = k \to$$

۲

$$k = \frac{1}{\sqrt{x}(-2x^2+x+1)} \to 6x^2-x+1 = -4x^2+2x+2 \to 10x^2-3x-1=0 \to$$

$$x = \frac{3 \pm \sqrt{49}}{20} \to \frac{1}{2}\ ✓ \quad -\frac{1}{4}\ ✗$$

$$f\left(\frac{1}{2}\right) = \frac{\frac{\sqrt{2}}{2}}{-\frac{1}{2}+\frac{1}{2}+1} = \frac{\sqrt{2}}{2}\ ✓$$

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۱۰)

$$fog(x) = \left(\frac{1}{\sqrt{x^2-1}}\left[\frac{1}{\sqrt{x^2-1}}\right]\right)^2 \to fog(x) = \left(\frac{2}{\sqrt{x^2-1}}\right)^2 = \frac{4}{x^2-1} = 4(x^2-1)^{-1}$$

$$x < \frac{\sqrt{5}}{2} \to x^2 < \frac{5}{4} \to x^2-1 < \frac{1}{4} \to \sqrt{x^2-1} < \frac{1}{2} \to \frac{1}{\sqrt{x^2-1}} > 2 \to \left[\frac{1}{\sqrt{x^2-1}}\right] = 2$$

۱

$$(fog)'(x) = -4(x^2-1)^{-2}(2x) \xrightarrow{x=\frac{\sqrt{5}}{2}} -4\left(\frac{1}{4}\right)^{-2}(\sqrt{5}) = -4\times16\times\sqrt{5} = -64\sqrt{5}$$

$$\frac{-64\sqrt{5}}{-48\sqrt{5}} = \boxed{\frac{4}{3}}$$

$$\left(fog\left(\tfrac{\sqrt{5}}{2}\right)\right)' = g'\left(\tfrac{\sqrt{5}}{2}\right)^- \times f'\left(g\left(\tfrac{\sqrt{5}}{2}\right)^-\right)$$

$$g(x) = (x^2-1)^{-\frac{1}{2}} \to g'(x) = -\frac{1}{2}(x^2-1)^{-\frac{3}{2}} \times 2x \to g'\left(\tfrac{\sqrt{5}}{2}^-\right) = \frac{1}{\sqrt{(\frac{5}{4})^- - 1}} = \frac{1}{\sqrt{(\frac{1}{4})^-}} = \frac{1}{(\frac{1}{2})^-} = 2^+$$

$$f'(2^+) = ((2x)^3)' = (8x^3)' = 24x^2 = 24\times4$$

$$\to g'\left(\tfrac{\sqrt{5}}{2}^-\right) \times f'\left(g\left(\tfrac{\sqrt{5}}{2}\right)^-\right) = -4\sqrt{5}\times4\times24 \to \frac{16\times4\times(-4\sqrt{5})}{-48\sqrt{5}} = 8$$

سوال ۸)

$$f(x) = 2\sqrt{x}(4x^2+3) = 8x^2\sqrt{x} + 6\sqrt{x} \to f'(x) = 20x\sqrt{x} + \frac{3}{\sqrt{x}} = \frac{20x^2+3}{\sqrt{x}}$$

$$y - 2\sqrt{\alpha}(4\alpha^2+3) = \frac{20\alpha^2+3}{\sqrt{\alpha}}(x-\alpha) \xrightarrow{(0,0)} -2\sqrt{\alpha}(4\alpha^2+3) = \frac{20\alpha^2+3}{\sqrt{\alpha}}(-\alpha)$$

$$\to 2(4\alpha^2+3) = 20\alpha^2+3 \to 12\alpha^2 = 3 \to \alpha^2 = \frac{1}{4}$$

$$m = \frac{20(\frac{1}{4})+3}{\sqrt{\frac{1}{2}}} = 8\sqrt{2}$$