۱)
$$f'(3)=m \longrightarrow \begin{matrix}(0,1)\\(3,5)\end{matrix} \longrightarrow \frac{5-1}{3-0}=\frac{4}{3} \longrightarrow f'(3)=\frac{4}{3}$$

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۲)
$$m=\frac{2-1}{2+1}=\frac{1}{3} \longrightarrow y-1=\frac{1}{3}(x+1) \longrightarrow y=\frac{1}{3}x+\frac{4}{3}$$

خط مماس: $\xrightarrow[\Delta=0]{\text{معادله مشترک}} \sqrt{ax-1}=\frac{1}{3}x+\frac{4}{3} \longrightarrow 9ax-9=x^2+8x+16 \longrightarrow$

$$x^2+(8-9a)x+25=0 \longrightarrow \Delta=(8-9a)^2-100=0 \longrightarrow 8-9a=10 \to a=-\frac{2}{9} \to f(5)=\sqrt{-\frac{10}{9}-1}$$ غ ق ق

$$\hookrightarrow 8-9a=-10 \to a=2 \to f(5)=3 \checkmark$$

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۳)
$$y'=\frac{(2x+m)(x+3)-(x^2+mx+1)}{(x+3)^2}=\frac{x^2+6x+3m-1}{(x+3)^2} \xrightarrow{x=1} \frac{6+3m}{16}=\frac{3}{4} \longrightarrow m=2$$

$$4y=3x+n \longrightarrow y=\frac{3}{4}x+\frac{n}{4} \longrightarrow \text{شیب خط}=\frac{3}{4}$$

$$y=\frac{x^2+2x+1}{x+3} \xrightarrow{x=1} y=\frac{4}{4}=1 \longrightarrow (1,1) \xrightarrow{\text{در معادله صدق می‌کند}}$$

$$4y-3x=n \longrightarrow 4-3=1=n \longrightarrow m+n=3$$

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۴)
$$f(x)=\frac{27-\sin^3x}{9-\sin^2x}=\frac{(3-\sin x)(9+\sin^2x+3\sin x)}{(3-\sin x)(3+\sin x)}=\frac{\sin^2x+3\sin x+9}{3+\sin x}$$

$$3g'\left(\frac{5\pi}{3}\right)-f'\left(\frac{5\pi}{3}\right)=(3g-f)'\left(\frac{5\pi}{3}\right)$$

$$(3g-f)(x)=\frac{9-\sin^2x-3\sin x-9}{3+\sin x}=-\sin x \to (3g-f)'(x)=-\cos x \to (3g-f)'\left(\frac{5\pi}{3}\right)=-\frac{\sqrt{3}}{2}$$

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۵)
$$g'(\sqrt[5]{3})\,f'(g(\sqrt[5]{3}))=(fog)'(\sqrt[5]{3}) \quad , \quad \underset{x>0}{g(x)}=\frac{1}{2x^5}$$

$$\underset{x>0}{fog(x)}=\frac{-1}{\sqrt[5]{\frac{1}{2x^5}+\frac{1}{2x^5}}}=\frac{-1}{\frac{1}{x}}=-x \longrightarrow (fog)'(x)=-1 \longrightarrow (fog)'(\sqrt[5]{3})=-1$$

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۶)
$$f(x)=xg(x)+1 \longrightarrow g(x)=\frac{f(x)-1}{x}$$

$$\lim_{x\to0}g(x)=\lim_{x\to0}\frac{f(x)-1}{x}=\lim_{x\to0}\frac{\left(\frac{-1+\sin x}{1+\sin x}\right)^2-1}{x} \longrightarrow \frac{\left(\frac{x-1}{x+1}\right)^2-1}{x}=\frac{-4x}{x(x^2+2x+1)}=\frac{-4}{(x+1)^2}=-4$$

---

۷)
$$\left.\begin{matrix}-y=x^2+1 \longrightarrow y=-x^2-1\\ y=k\end{matrix}\right\} \longrightarrow k=-x^2-1 \longrightarrow x^2=-k-1 \longrightarrow x=\pm\sqrt{-k-1}$$

$$y'=-2x \longrightarrow x=\sqrt{-k-1} \longrightarrow y'=-2\sqrt{-k-1}$$
$$\hookrightarrow x=-\sqrt{-k-1} \longrightarrow y'=2\sqrt{-k-1}$$
$$\longrightarrow -4(-k-1)=-1 \longrightarrow -k-1=\frac{1}{4} \longrightarrow k=-\frac{5}{4}$$

$$y=-\frac{5}{4} \longrightarrow \text{فاصله از مبدا}=\frac{5}{4}$$

(۸

$f(x)$ و محور xها نقطه تقاطع $=(0,0) \rightarrow f'(0)=$ شیب خط

$$f'(x)=\frac{۲}{۲\sqrt{x}}(۴x^۲+۳)+۲\sqrt{x}\,(۸x) \longrightarrow f'(0)=\frac{1}{0}(۳)+0=\frac{۳}{0}$$ موازی محور y

شیب خط: تعریف نشده

(۹

خط: $y=kx$

$$f(x)=\frac{\sqrt{x}}{-۲x^۲+x+1} \Bigg\} \rightarrow kx=\frac{\sqrt{x}}{-۲x^۲+x+1} \rightarrow k=\frac{1}{\sqrt{x}(-۲x^۲+x+1)}$$

$y'=k$

$$f'(x)=\frac{\frac{1}{۲\sqrt{x}}(-۲x^۲+x+1)-\sqrt{x}(-۴x+1)}{(-۲x^۲+x+1)^۲}=\frac{۶x^۲-x+1}{۲\sqrt{x}(-۲x+x+1)^۲}=k \longrightarrow$$

$$k=\frac{1}{\sqrt{x}(-۲x^۲+x+1)} \longrightarrow ۶x^۲-x+1=-۴x^۲+۲x+۲ \rightarrow ۱۰x^۲-۳x-1=0 \longrightarrow$$

$$x=\frac{۳\pm\sqrt{۴۹}}{۲۰} \rightarrow \frac{1}{۲} \checkmark \qquad -\frac{1}{۴} \times \longrightarrow$$

$$f\left(\frac{1}{۲}\right)=\frac{\frac{\sqrt{۲}}{۲}}{-\frac{1}{۲}+\frac{1}{۲}+1}=\frac{\sqrt{۲}}{۲}$$

(۱۰

$$fog(x)=\left(\frac{1}{\sqrt{x^۲-1}}\left[\frac{1}{\sqrt{x^۲-1}}\right]\right)^۲ \longrightarrow fog(x)=\left(\frac{۲}{\sqrt{x^۲-1}}\right)^۲=\frac{۴}{x^۲-1}=۴(x^۲-1)^{-1}$$

$$x<\frac{\sqrt{۵}}{۲} \rightarrow x^۲<\frac{۵}{۴} \rightarrow x^۲-1<\frac{1}{۴} \rightarrow \sqrt{x^۲-1}<\frac{1}{۲} \rightarrow \frac{1}{\sqrt{x^۲-1}}>۲ \rightarrow \left[\frac{1}{\sqrt{x^۲-1}}\right]=۲$$

$$(fog)'(x)=-۴(x^۲-1)^{-۲}(۲x) \xrightarrow{x=\frac{\sqrt{۵}}{۲}} -۴\left(\frac{1}{۴}\right)^{-۲}(\sqrt{۵})=-۴\times۱۶\times\sqrt{۵}=-۶۴\sqrt{۵}$$

$$\boxed{\frac{-۶۴\sqrt{۵}}{-۴۸\sqrt{۵}}=\frac{۴}{۳}}$$