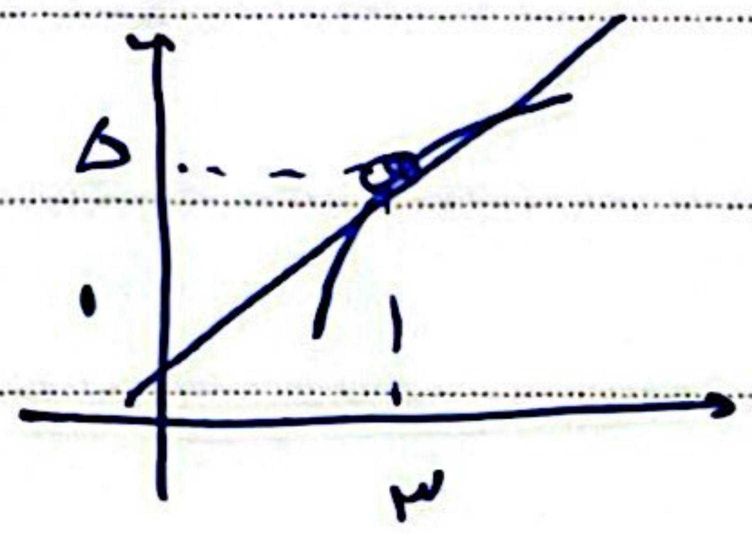
فاطمه سلطان آبادی



(۱) [illegible] $\Rightarrow (3,5)\ ,\ (0,1)$

$a = \frac{1-5}{0-3} = \frac{-4}{-3} = \frac{4}{3}$

$\Rightarrow f'(3) = \frac{4}{3}$

(۳) $m = \frac{\text{قرینه ضریب } x}{\text{ضریب } y} \Rightarrow m = \frac{3}{4}$

$g' = \frac{(2x+m)(x+3) - (x^2+mx+1)(1)}{(x+3)^2}$

$\xrightarrow{x=1} g' = \frac{3(2+m)}{16} = \frac{3}{4} \Rightarrow 2+m = 4$

$\Rightarrow m = 2$

$x = 1 \Rightarrow g = \frac{2+m}{4} = 1 \Rightarrow 4(1) - 3(1) = n \Rightarrow n = 1$

$\Rightarrow m+n = 3$

(۲) $(2,2),(-1,1) \Rightarrow m = \frac{2-1}{2+1} = \frac{1}{3}$

$\Rightarrow y - 1 = \frac{1}{3}(x+1)$

$\Rightarrow y = \frac{x}{3} + \frac{4}{3}$

$\Delta = (8-9a)^2 - 4(1)(25) = 0$ $\quad a = -\frac{9}{2}$

$\Rightarrow (8-9a)^2 = 100 \quad 8-9a = -10$

$\Rightarrow 8-9a = 10 \Rightarrow a = 2$ ✓

$\Delta = 0 \Leftarrow$ مماس

$(\sqrt{ax-1})^2 = (\frac{x}{3} + \frac{4}{3})^2 \Rightarrow ax - 1 = \frac{x^2+8x+16}{9} \Rightarrow x^2 + (8-9a)x + 25 = 0$

$a = 2 \Rightarrow f(x) = \sqrt{2x-1} \Rightarrow f(5) = \sqrt{9} = 3$

(9)

$g = x^2 + 1 \xrightarrow{\text{قرینه}} f(x) = -(x^2+1)$

$\Rightarrow f'(x) = -2x$

$f'(a) \times f'(-a) = -1$

$\Rightarrow (-2a)(2a) = -1 \Rightarrow a^2 = \frac{1}{4} \xrightarrow{a>0} a = \frac{1}{2}$

$\Rightarrow f(\frac{1}{2}) = -((\frac{1}{2})^2 + 1) = \frac{-5}{4}$

$\Rightarrow A(\frac{1}{2}, \frac{-5}{4})$

عرض نقطه $y = \frac{-5}{4}$

$\Rightarrow d = \frac{5}{4}$

(10)

$f \circ g(\frac{\sqrt{5}}{2}) = g'(\frac{\sqrt{5}}{2}) \cdot f'(g(\frac{\sqrt{5}}{2}))$

$g(x) = (x^2-1)^{-\frac{1}{2}} \Rightarrow g'(x) = -\frac{1}{2}(x^2-1)^{-\frac{3}{2}}(2x)$

$\Rightarrow g'(\frac{\sqrt{5}}{2}) = -\frac{1}{2} \times (\frac{5}{4} - 1)^{-\frac{3}{2}} \times \sqrt{5} = -\frac{1}{2} \times 8 \cdot \sqrt{5}$

$= -4\sqrt{5}$

$g(\frac{\sqrt{5}}{2}) = \frac{1}{\sqrt{\frac{5}{4} - 1}} = \frac{1}{\sqrt{\frac{1}{4}}} = \frac{1}{(\frac{1}{2})} = 2$

$f'(2) \Rightarrow ((2x)^3)' = (8x^3)' = 24x^2 = 24 \times 4$

$\Rightarrow 24 \times 4 \times -4\sqrt{5}$ (1)

$-384\sqrt{5}$

$$g'(\sqrt[5]{3}) \times f'(g(\sqrt[5]{3})) = (f \circ g)'(\sqrt[5]{3})$$ (5

$$x = \sqrt[5]{3} > 0 \Rightarrow f(x) = -\frac{1}{\sqrt[5]{2x}}$$

$$g(x) = \frac{1}{2x^5}$$

$$\Rightarrow f \circ g(x) = -\frac{1}{\sqrt[5]{\frac{2}{2x^5}}} = -x \Rightarrow (f \circ g)'(x) = -1$$

(6

$$g(x) = \frac{f(x) - 1}{x} \Rightarrow \lim_{x \to 0} g(x) = \lim_{x \to 0} \frac{f(x) - 1}{x} = f'(0)$$

$$f'(x) = 2\left(\frac{\sin x - 1}{\sin x + 1}\right)\left(\frac{\cos x(\sin x + 1 - \sin x + 1)}{(\sin x + 1)^2}\right)$$

$$\Rightarrow f'(0) = 2(-1)(2) = -4$$

(7)

$$f(x) = 8x^{\frac{5}{2}} + 4x^{\frac{1}{2}} \Rightarrow f'(x) = 20x^{\frac{3}{2}} + 3x^{-\frac{1}{2}}$$

$$\lim_{x \to a} \frac{f(x) - 0}{x - a} = f'(a) \rightarrow 8a^{\frac{5}{2}} + 4a^{\frac{1}{2}} = 20a^{\frac{3}{2}} + 3a^{-\frac{1}{2}}$$

$$\Rightarrow 3a^{\frac{1}{2}} = 12a^{\frac{5}{2}} \Rightarrow 1 = 4a^2 \rightarrow a = \frac{1}{2}$$

$$f'(a) = f'\left(\frac{1}{2}\right) = 20 \times \frac{1}{2\sqrt{2}} + 3\sqrt{2}$$

$$= 5\sqrt{2} + 3\sqrt{2} = 8\sqrt{2}$$