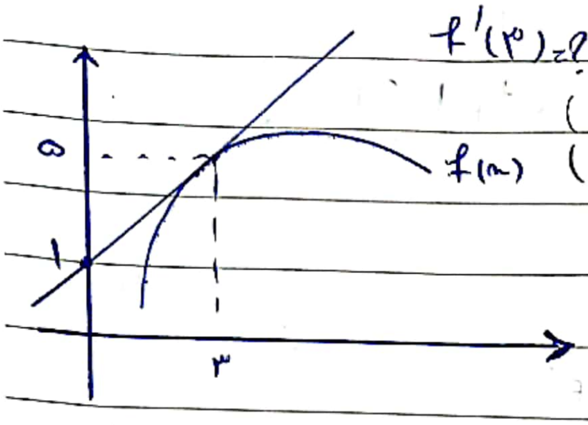
«دارو‌های ... A...»　　　　　　　　　　　　　　　　　　　　　　　　　۱۷/۷۵　　　　　　　　　　　　　　　　　　　تمرین‌های الف الحسنی»

۱-



$f'(3)=?$

$(0,1)$ $(3,5)$ $\Rightarrow m=\frac{4}{3} \Rightarrow f'(3)=\frac{4}{3}$

۲

۲- $f(x)=\sqrt{ax-1}$　　$(-1,1)$　$(2,2) \Rightarrow m=\frac{1}{3}$

$F(x)=g(x)$　　$y-1=\frac{1}{3}(x+1) \Rightarrow y=\frac{1}{3}x+\frac{4}{3}$　　۲

$\sqrt{ax-1}=\frac{1}{3}x+\frac{4}{3}$　　$9ax-9=x^2+8x+16$　　$\Rightarrow f(x)=\sqrt{2x-1}$

$3\sqrt{ax-1}=x+4$　　$x^2+(8-9a)x+25=0$　　$f(5)=\sqrt{10-1}$

$8-9a=10$ یا $8-9a=-10$

$9a=-2$　　$18=9a$　　$\sqrt{9}=3$

$a=\frac{-2}{9}$ ✗　　$\boxed{a=2}$

۳- $f(x)=\frac{x^2+mx+1}{x+3}$　　$4y-3x=n$　　$m+n=?$

$g(x)=\frac{n+3x}{4}$

$f'(x)=\frac{x^2+6x+3m-1}{(x+3)^2}$　　$g'(x)=\frac{3}{4}$

۲

$f'(1)=g'(1) \Rightarrow \frac{6+3m}{16}=\frac{3}{4} \Rightarrow 3m+6=12 \Rightarrow \boxed{m=2}$

$f(1)=g(1) \Rightarrow 1=\frac{3+n}{4} \Rightarrow 3+n=4 \Rightarrow n=1 \Rightarrow \boxed{m+n=3}$

۴- $f(x)=\frac{27-\sin^3 x}{9-\sin^2 x}$　　$g(x)=\frac{3}{3+\sin x}$

$3g'\left(\frac{5\pi}{3}\right)-f'\left(\frac{5\pi}{3}\right)=(3g-f)'\left(\frac{5\pi}{3}\right)$

$f(x)=\frac{\cancel{(3-\sin x)}(\sin^2 x+3\sin x+9)}{\cancel{(3-\sin x)}(3+\sin x)}$　　۲

$\frac{9}{(3+\sin x)}-\frac{\sin^2 x+3\sin x+9}{(3+\sin x)}=\frac{-\sin x\cancel{(\sin x+3)}}{\cancel{(\sin x+3)}}$

$(3g-f)'(x)=-\cos x \Rightarrow -\cos\frac{5\pi}{3}=\boxed{-\frac{1}{2}}$

$fog(x) = -x \longrightarrow (fog)'(x) = -1 \longrightarrow (fog)'(\sqrt[5]{3}) = -1$

ـ۵

$f(x) = \dfrac{1}{\sqrt[5]{x + |x|}} \qquad g(x) = \dfrac{1}{(x^5 + |x^5|)\,2x^5}$

۱,۷۵

$g'(\sqrt[5]{3})\, f'(g(\sqrt[5]{3})) = (fog(x))' = \boxed{1}$

$f(g(x)) = \dfrac{1}{\sqrt[5]{\left|\dfrac{1}{2x^5}\right| + \left|\dfrac{1}{2x^5}\right|}} = \dfrac{1}{\sqrt[5]{\dfrac{\cancel{2}}{\cancel{2}x^5}}} = \dfrac{1}{\sqrt[5]{\dfrac{1}{x^5}}} = \dfrac{1}{\dfrac{1}{x}} = x$

$\Rightarrow fog(x) = x \Rightarrow (fog(x))' = \boxed{1}$

ـ۶

$f(x) = \left(\dfrac{\sin x - 1}{\sin x + 1}\right)^2 \qquad f(x) = x\,g(x) + 1$

$\lim g(x) = ?$

$f'(x) = g(x) + g'(x)\,x \qquad f'(x) = 2\left(\dfrac{\sin x - 1}{\sin x + 1}\right)\left(\dfrac{2}{(\sin x + 1)^2}\right) \times \cos x \quad g(0) = ?$

$f'(0) = g(0)$

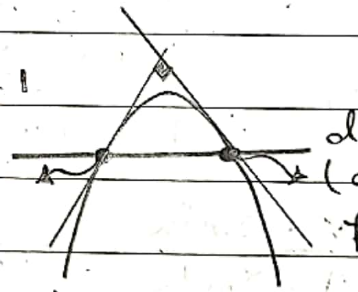
$f'(0) = 2(-1)\left(\dfrac{2}{1}\right) \times 1 = -4 \Rightarrow \boxed{g(0) = -4}$

۲

ـ۷

$y = x^2 + 1 \xrightarrow{\text{قرینه نسبت به } x} y = -x^2 - 1$

$y' = -2x$



$f'(-\alpha) = 2\alpha \qquad f'(\alpha) = -2\alpha$

$(2\alpha)(-2\alpha) = -1 \longleftarrow mm' = -1 \longleftarrow$ دو خط عمودند

$+4\alpha^2 = +1 \longrightarrow \alpha^2 = \dfrac{1}{4} \Rightarrow \alpha = \pm\dfrac{1}{2} \Rightarrow -\alpha^2 - 1 = -\dfrac{1}{4} - 1 = -\dfrac{5}{4}$

$\Rightarrow d: y = \dfrac{-5}{4}$

فاصله خط d از مبدأ مختصات: $\dfrac{5}{4}$

۲

۸-

$x \geq 0$ *

$$f(x) = 2\sqrt{x}\,(4x^2+3)$$

$$f'(x) = \underbrace{\frac{2}{2\sqrt{x}}(4x^2+3)}_{4x+\frac{3}{\sqrt{x}}} + \underbrace{(8x)(2\sqrt{x})}_{16x\sqrt{x}} = 20x\sqrt{x} + \frac{3}{\sqrt{x}}$$

$(\alpha, d\alpha)$

$$f(\alpha) = 8\alpha^2\sqrt{\alpha} + 6\sqrt{\alpha} = d\alpha$$

$$f'(\alpha) = 20\alpha\sqrt{\alpha} + \frac{3}{\sqrt{\alpha}} = d \xrightarrow{\times\alpha} 20\alpha^2\sqrt{\alpha} + 3\sqrt{\alpha} = d\alpha$$

$$d\alpha = 20\alpha^2\sqrt{\alpha} + 3\sqrt{\alpha} = 8\alpha^2\sqrt{\alpha} + 6\sqrt{\alpha}$$

$$12\alpha^2\cancel{\sqrt{\alpha}} = 3\cancel{\sqrt{\alpha}} \to 4\alpha^2 = 1 \to \alpha^2 = \frac{1}{4} \to \alpha = \pm\frac{1}{2} \Rightarrow \alpha = +\frac{1}{2}$$

$$f'(+\tfrac{1}{2}) = 8(\tfrac{1}{2})^2\sqrt{\tfrac{1}{2}} + 6\sqrt{\tfrac{1}{2}} = \frac{d}{2} \to \frac{8}{\sqrt{2}} = \frac{d}{2} \to d = 8\sqrt{2}$$ ✓

۲

۹- ⓪

$$f(x) = \frac{\sqrt{x}}{-2x^2+x+1}$$

x — d خط $\to y = ax$ $\qquad A(\alpha, a\alpha)$

$$f(\alpha) = \frac{\sqrt{\alpha}}{-2\alpha^2+\alpha+1} = a\alpha \to a\sqrt{\alpha}(-2\alpha^2+\alpha+1) = 1 \to -2a\alpha^{\frac{5}{2}} + a\alpha^{\frac{3}{2}} + a\alpha^{\frac{1}{2}} = 1$$

$$\xrightarrow{\text{مشتق}} -5a\alpha^{\frac{3}{2}} + \frac{3}{2}a\alpha^{\frac{1}{2}} + \frac{1}{2}a\alpha^{-\frac{1}{2}} = 0 \xrightarrow[\times 2\sqrt{\alpha}]{\div a} -10\alpha^2 + 3\alpha + 1 = 0 \to \begin{cases}\alpha = -\frac{1}{5}\\ \alpha = \frac{1}{2}\end{cases}$$

$$f(\alpha) = \frac{\sqrt{\frac{1}{2}}}{-2(\frac{1}{2})^2 + \frac{1}{2} + 1} = \frac{\sqrt{2}}{2}$$

۱۰-

$$f(x) = (x[x])^3 \qquad g(x) = \frac{1}{\sqrt{x^2-1}} = (x^2-1)^{-\frac{1}{2}}$$

$$\frac{(f\circ g)'_-(\frac{\sqrt{5}}{2})}{-48\sqrt{5}} = ? \qquad g'(\tfrac{\sqrt{5}}{2})\, f'(\underbrace{g(\tfrac{\sqrt{5}}{2})}_{2^+})$$

$$g'(x) = -\frac{1}{\cancel{2}}(x^2-1)^{-\frac{3}{2}} \times \cancel{2}x = \frac{-x}{\sqrt{(x^2-1)^3}} = \frac{-\frac{\sqrt{5}}{2}}{\frac{1}{8}} = -4\sqrt{5}$$

$$f(2^+) = (2x)^3 \Rightarrow 3(2x)^2 \times 2 = 24x^2 = 24 \times 4 = 96$$

$$\Rightarrow (f\circ g)' = -96 \times 4\sqrt{5}$$

$$\Rightarrow \frac{\cancel{-96} \times 4\cancel{\sqrt{5}}}{\cancel{-48}\cancel{\sqrt{5}}} = \boxed{8}$$ ✓

۲