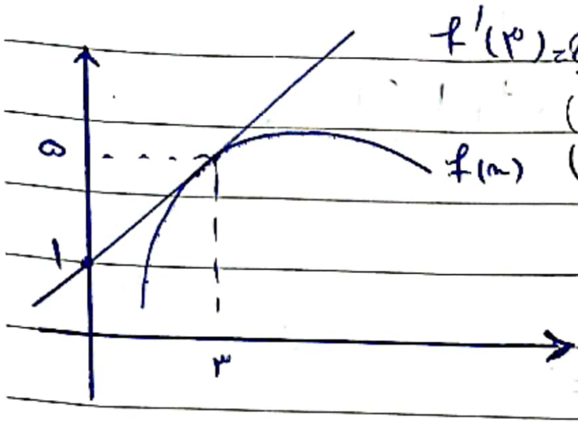
«دوازدهم، دفتر جمع‌بندی A» «تمرین‌های الف الحسنی»

۱-



$f'(3)=?$

$(0,1)\quad (3,5) \Rightarrow m=\frac{4}{3} \Rightarrow \boxed{f'(3)=\frac{4}{3}}$

۲-

$f(x)=\sqrt{ax-1}\qquad (-1,1)\quad (2,2) \Rightarrow m=\frac{1}{3}$

$f(x)=g(x)\qquad y-1=\frac{1}{3}(x+1) \Rightarrow y=\frac{1}{3}x+\frac{4}{3}$

$\sqrt{ax-1}=\frac{1}{3}x+\frac{4}{3}$

$3\sqrt{ax-1}=x+4 \to 9ax-9=x^2+8x+16$

$x^2+(8-9a)x+25=0$

$8-9a=10$ یا $8-9a=-10$

$9a=-2 \qquad 18=9a$

$a=\frac{-2}{9}$ یا $\boxed{a=2}$ ✓

$\Rightarrow f(x)=\sqrt{2x-1}$

$f(5)=\sqrt{10-1}$

$\sqrt{9}=3$ ✓

۳-

$f(x)=\frac{x^2+mx+1}{x+3}\qquad 4y-3x=n\qquad m+n=?$

$g(x)=\frac{n+3x}{4}$

$f'(x)=\frac{x^2+6x+3m-1}{(x+3)^2}\qquad g'(x)=\frac{3}{4}$

$f'(1)=g'(1) \Rightarrow \frac{6+3m}{16}=\frac{3}{4} \Rightarrow 3m+6=12 \Rightarrow \boxed{m=2}$

$f(1)=g(1) \Rightarrow 1=\frac{3+n}{4} \Rightarrow 3+n=4 \Rightarrow n=1 \Rightarrow \boxed{m+n=3}$

۴-

$f(x)=\frac{27-\sin^3 x}{9-\sin^2 x}\qquad g(x)=\frac{3}{3+\sin x}$

$3g'(\frac{5\pi}{3})-f'(\frac{5\pi}{3})=(3g-f)'(\frac{5\pi}{3})$

$f(x)=\frac{\cancel{(3-\sin x)}(\sin^2 x+3\sin x+9)}{\cancel{(3-\sin x)}(3+\sin x)}$

$\frac{\cancel{9}}{(3+\sin x)}-\frac{\sin^2 x+3\sin x+\cancel{9}}{(3+\sin x)}=\frac{-\sin x(\cancel{\sin x+3})}{(\cancel{\sin x+3})}$

$(3g-f)'(x)=-\cos x \Rightarrow -\cos\frac{5\pi}{3}=\boxed{\frac{-1}{2}}$

۵

$f(x) = \frac{1}{\sqrt[5]{x+|x|}}$ $\qquad$ $g(x) = \frac{1}{(x^5+|x^5|)\,2x^5}$

$g'(\sqrt[5]{3})\, f'(g(\sqrt[5]{3})) = (f\circ g(x))' = \boxed{1}$

$f(g(x)) = \frac{1}{\sqrt[5]{\frac{1}{2x^5}+\left|\frac{1}{2x^5}\right|}} = \frac{1}{\sqrt[5]{\frac{\cancel{2}}{\cancel{2}x^5}}} = \frac{1}{\sqrt[5]{\frac{1}{x^5}}} = \frac{1}{\frac{1}{x}} = x$

$\Rightarrow f\circ g(x) = x \Rightarrow (f\circ g(x))' = \boxed{1}$

۶

$f(x) = \left(\frac{\sin x-1}{\sin x+1}\right)^2$ $\qquad$ $f(x) = x\,g(x)+1$ $\qquad$ $\lim g(x) = ?$

$f'(x) = g(x)+g'(x)\,x$ $\qquad$ $f'(x) = 2\left(\frac{\sin x-1}{\sin x+1}\right)\left(\frac{2}{(\sin x+1)^2}\right)\times\cos x$ $\qquad$ $g(0)=?$

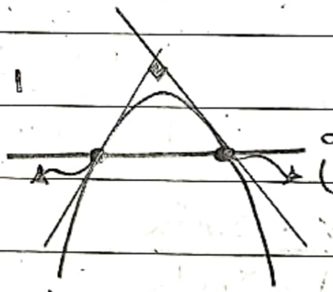
$f'(0) = g(0)$

$f'(0) = 2(-1)\left(\frac{2}{1}\right)\times 1 = -4 \Rightarrow \boxed{g(0) = -4}$

۷

$y = x^2+1 \xrightarrow{\text{قرینه نسبت به } x\text{ها}} y = -x^2-1$

$y' = -2x$



$(-\alpha, -\alpha^2-1)$ $\qquad$ d $\qquad$ $(\alpha, -\alpha^2-1)$

$f'(-\alpha) = 2\alpha$ $\qquad$ $f'(\alpha) = -2\alpha$

$(2\alpha)(-2\alpha) = -1 \leftarrow mm' = -1 \leftarrow$ چون عمودند

$+4\alpha^2 = +1 \to \alpha^2 = \frac{1}{4} \Rightarrow \alpha = \pm\frac{1}{2} \Rightarrow -\alpha^2-1 = -\frac{1}{4}-1 = -\frac{5}{4}$

$\Rightarrow d: y = -\frac{5}{4}$

فاصله خط d از مبدأ مختصات: $\frac{5}{4}$

۸-

$x \geq 0$ *

$f(x) = 2\sqrt{x}\,(4x^2+3)$

$f'(x) = \underbrace{\frac{2}{2\sqrt{x}}(4x^2+3)}_{4x+\frac{3}{\sqrt{x}}} + \underbrace{(8x)(2\sqrt{x})}_{16x\sqrt{x}} = 20x\sqrt{x} + \frac{3}{\sqrt{x}}$ $(\alpha, d\alpha)$

$f(\alpha) = 8\alpha^2\sqrt{\alpha} + 6\sqrt{\alpha} = d\alpha$

$f'(\alpha) = 20\alpha\sqrt{\alpha} + \frac{3}{\sqrt{\alpha}} = d \xrightarrow{\times\alpha} 20\alpha^2\sqrt{\alpha} + 3\sqrt{\alpha} = d\alpha$

$d\alpha = 20\alpha^2\sqrt{\alpha} + 3\sqrt{\alpha} = 8\alpha^2\sqrt{\alpha} - 6\sqrt{\alpha}$

$12\alpha^2\sqrt{\alpha} = 3\sqrt{\alpha} \rightarrow 4\alpha^2 = 1 \rightarrow \alpha^2 = \frac{1}{4} \rightarrow \alpha = \pm\frac{1}{2} \Rightarrow \alpha = +\frac{1}{2}$

$f'(+\frac{1}{2}) = 8(\frac{1}{2})^2\sqrt{\frac{1}{2}} + 6\sqrt{\frac{1}{2}} = \frac{d}{2} \rightarrow \frac{8}{\sqrt{2}} = \frac{d}{2} \rightarrow d = 8\sqrt{2}$

۹-

$f(x) = \frac{\sqrt{x}}{-2x^2+x+1}$

x

۱۰-

$f(x) = (x[x])^3 \qquad g(x) = \frac{1}{\sqrt{x^2-1}} = (x^2-1)^{-\frac{1}{2}}$

$\frac{(f\circ g)'_-(\frac{\sqrt{5}}{2})}{-48\sqrt{5}} = ? \rightsquigarrow g'(\frac{\sqrt{5}}{2})\, f'(\underbrace{g(\frac{\sqrt{5}}{2})}_{2^+})$

$g'(x) = \frac{-1}{2}(x^2-1)^{-\frac{3}{2}} \times 2x = \frac{-x}{\sqrt{(x^2-1)^3}} = \frac{-\frac{\sqrt{5}}{2}}{\frac{1}{8}} = -4\sqrt{5}$

$f(2^+) = (2x)^3 \Rightarrow 3(2x)^2 \times 2 = 24x^2 = 24 \times 4 = 96$

$\Rightarrow (f\circ g)' = -96 \times 4\sqrt{5}$

$\Rightarrow \frac{-96 \times 4\sqrt{5}}{-48\sqrt{5}} = \boxed{8}$