(سارا امینی راد) ۱۴

۱
$$m = \frac{5-1}{3-0} = \frac{4}{3} \qquad f'(5) = \frac{4}{3}$$
(۲)

۲
$$m = \frac{2-1}{2-(-1)} = \frac{1}{3} \qquad f(x) = \sqrt{ax-1} \xrightarrow[\text{خط مماس}]{\text{شیب}} f'(x) = \frac{a}{2\sqrt{ax-1}} \to \frac{a}{2\sqrt{ax-1}} = \frac{1}{3} \to 3a = 2\sqrt{ax-1} \quad (I)$$

$$m = \frac{2-1}{2-(-1)} = \frac{1}{3} \qquad y - 1 = \frac{1}{3}(x+1) \Rightarrow y = \frac{1}{3}x + \frac{4}{3}$$
(۱)

معادله خط گذرنده از ۲ نقطه $= y = \frac{1}{3}x + \frac{4}{3} \to 3y = x+4 \to x+4 = 3\sqrt{ax-1} \quad (II)$

$$f'(x) = \frac{a}{2\sqrt{ax-1}} = \frac{1}{3} \quad \text{در نقطه تماس} \Rightarrow 3a = 2\left(\frac{1}{3}x + \frac{4}{3}\right)$$

$$\xrightarrow{I, II} x+4 = \left(\frac{3a}{2}\right)^2 = \frac{9a}{2} \to x = 4.5a - 4$$

$$ax - 1 = \left(\frac{1}{3}x + \frac{4}{3}\right)^2 \Rightarrow a = 1 \qquad f(5) = \sqrt{5-1} = 2$$

$$\xrightarrow{II} 4.5a - 4 + 4 = 3\sqrt{a(4.5a-4)-1} \to 9a^2 - 16a - 4 = 0 \to \begin{cases} a = 2 \\ a = -\frac{2}{9} \end{cases} \qquad f(5) = \sqrt{2(5)-1} = \sqrt{9} = 3$$

۳
$$y = \frac{3}{4}x + \frac{n}{4}$$

$$\frac{\text{مشتق}}{\text{شیب}} = \frac{3}{4} \qquad \text{مشتق تابع} \to \frac{(2x+m)(x+3) - (x^2+mx+1)}{(x+3)^2}$$

$$\Rightarrow y(1) = \frac{3}{4} \qquad (2+m)(4) - (m+2) \Rightarrow 6 + 3m$$
(۲)

$$\text{مخرج} \quad (1+3)^2 = 16 \qquad y(1) = \frac{6+3m}{16} = \frac{3}{4} \to m = 2$$

$$y(1) = \frac{4}{4} = 1 \qquad (1,1) \Rightarrow 4(1) - 3(1) = n \to n = 1 \qquad m + n = 2 + 1 = 3$$

۴
$$g(x) = 3(3+\sin x)^{-1} = 3(-1)(3+\sin x)^{-2}\cos x \to g(x) = \frac{-3\cos x}{(3+\sin x)^2}$$
(۱)

$$3g'\left(\frac{5\pi}{3}\right) - f'\left(\frac{5\pi}{3}\right) = (3g(x) - f(x))'\left(\frac{5\pi}{3}\right)$$

$$g = \frac{-3\left(\frac{1}{2}\right)}{\left(3 - \frac{\sqrt{3}}{2}\right)^2} \Rightarrow 3g = -\frac{18}{(6-\sqrt{3})^2}$$

$$\to (3g - f)(x) = \left(\frac{9}{3+\sin x} - \frac{27 - \sin^3 x}{9 - \sin^2 x}\right) = \frac{9}{3+\sin x} - \frac{(3-\sin x)(9+\sin^2 x + 3\sin x)}{(3-\sin x)(3+\sin x)} = -\sin x$$

$$\frac{\text{مشتق}}{\text{صورت}} = -3\sin^2 x\cos x = u \qquad \frac{u'v - uv'}{v^2} \Rightarrow f\left(\frac{5\pi}{3}\right) = 0 \qquad 3g - f \Rightarrow -\frac{18}{(6-\sqrt{3})^2}$$

$$\text{مخرج} \quad = -2\sin x\cos x = v \qquad \to (3g - f)'(x) = -\cos x \to (3g-f)'\left(\frac{5\pi}{3}\right) = -\cos\left(\frac{5\pi}{3}\right) = \frac{-1}{2}$$

۵
$$x + |x| = 2x \qquad f(x) = -(2x)^{-\frac{1}{5}}$$

$$x^5 + |x^5| = 2x^5 \qquad g(x) = \frac{1}{2x^5} = \frac{1}{2}x^{-5}$$

$$g'(x) = \frac{1}{2}(-5)x^{-6} = -\frac{5}{2}x^{-6} \Rightarrow x^5 = 3x \qquad g = -\frac{5}{2(3x)} = \frac{-5}{6\sqrt[5]{3}}$$
(۲)

$$g(\sqrt[5]{3}) = \frac{1}{6}$$

$$f(x) = -(2x)^{-\frac{1}{5}} \to -\left(-\frac{1}{5}\right)(2x)^{-\frac{6}{5}} \times 2 \to f(x) = \frac{2}{5}(2x)^{-\frac{6}{5}}$$

$$x = \frac{1}{6} \qquad f\left(\frac{1}{6}\right) = \frac{2}{5}3^{\frac{6}{5}} \qquad \to \frac{-5}{6\sqrt[5]{3}} \times \frac{2}{5}3^{\frac{6}{5}} = -1$$

(سارا السی راد)

6

$$g(x)=\frac{f(x)-1}{x}\qquad h(x)\xrightarrow{\text{مشتق}}\frac{(\cos x)(1+\sin x)-(-1+\sin x)(\cos x)}{(1+\sin x)^2}$$

$$\lim g(x)=\lim\frac{f(x)-1}{x}\qquad =\frac{2\cos x}{(1+\sin x)^2}$$

$$\lim g(x)=f'(0)$$ 2

$$f'(x)=2h'(x)h(x)\qquad h(0)=-1\qquad f'(0)=2(-1)(2)=-4$$

$$h'(0)=2$$

7

$$y=x^2+1\qquad x^2+1=c\qquad x^2=c-1\qquad x=\pm\sqrt{c-1}$$

$$m=2x\qquad m_1=2\sqrt{c-1}\qquad m_2=-2\sqrt{c-1}$$ 2

$$m_1m_2=-1\qquad (2\sqrt{c-1})(-2\sqrt{c-1})=-1\qquad -4(c-1)=-1\qquad c=\frac{5}{4}$$

8 ۰٫۵

$$f(x)=8x^{\frac{5}{2}}+4x^{\frac{1}{2}}\Rightarrow\frac{16}{7}x^{\frac{7}{2}}+4x^{\frac{3}{2}}\qquad m=\frac{20(\frac{1}{2})+3}{\sqrt{\frac{1}{2}}}=8\sqrt{2}$$

$$\frac{16a^{\frac{7}{2}}+4a^{\frac{3}{2}}}{a}=\frac{16}{7}a^{\frac{5}{2}}+4a^{\frac{1}{2}}\qquad a^{\frac{1}{2}}\left(\frac{40}{7}a^2+2\right)=0$$

$$\to a=0\qquad m=f'(0)=0$$

$$f(\alpha)=4\sqrt{\alpha}(2\alpha^2+1)=8\alpha^2\sqrt{\alpha}+4\sqrt{\alpha}\to f'(\alpha)=20\alpha\sqrt{\alpha}+\frac{2}{\sqrt{\alpha}}=\frac{20\alpha^2+2}{\sqrt{\alpha}}$$

$$y-4\sqrt{\alpha}(2\alpha^2+1)=\frac{20\alpha^2+2}{\sqrt{\alpha}}(x-\alpha)\xrightarrow{(0,0)}-4\sqrt{\alpha}(2\alpha^2+1)=\frac{20\alpha^2+2}{\sqrt{\alpha}}(-\alpha)$$

$$\to 4(2\alpha^2+1)=20\alpha^2+2\to 12\alpha^2=2\to\alpha^2=\frac{1}{6}$$

خط $\to y=ax\qquad A(\alpha,a\alpha)$

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$$f'(x)=\frac{1}{2\sqrt{x}}(-2x^2+x+1)-\sqrt{x}(-4x+1)\qquad\frac{f(a)}{a}=f'(a)=\frac{1}{\sqrt{a}(-2a^2+a+1)}$$

$$f(\alpha)=\frac{\sqrt{\alpha}}{-2\alpha^2+\alpha+1}=a\alpha\to a\sqrt{\alpha}(-2\alpha^2+\alpha+1)=1\to -2a\alpha^{\frac{5}{2}}+a\alpha^{\frac{3}{2}}+a\alpha^{\frac{1}{2}}=1$$

$$-2a+1=0\to a=\frac{1}{2}\qquad 4a^2-4a+1=0\qquad (2a-1)^2=0\qquad\to a=\frac{1}{2}$$

$$\xrightarrow{\text{مشتق}}-5a\alpha^{\frac{3}{2}}+\frac{3}{2}a\alpha^{\frac{1}{2}}+\frac{1}{2}a\alpha^{-\frac{1}{2}}=0\xrightarrow[\times 2\sqrt{\alpha}]{\div a}-10\alpha^2+3\alpha+1=0\to\begin{cases}\alpha=-\frac{1}{5}\\ \alpha=\frac{1}{2}\end{cases}$$

$$f(\alpha)=\frac{\sqrt{\frac{1}{2}}}{-2(\frac{1}{2})^2+\frac{1}{2}+1}=\frac{\sqrt{2}}{2}$$

مشتق چپ

$$x=\frac{\sqrt{5}}{2}=1.118\qquad [x]=1\qquad f(x)=x^3$$

$$\left(f\circ g\left(\frac{\sqrt{5}}{2}\right)\right)'=g'\left(\frac{\sqrt{5}}{2}^-\right)\times f'\left(g\left(\frac{\sqrt{5}}{2}\right)^-\right)$$

10 1

$$f\circ g(x)=f(g(x))\Rightarrow\frac{1}{\sqrt{\frac{5}{4}-1}}=2\qquad f(g(x))=(g(x))^3$$

$$f\circ g=(g(x))^3=3g(x)^2g'(x)\qquad g'(x)=\frac{-x}{(x^2-1)^{\frac{3}{2}}}\qquad g'(x)=\frac{-\frac{\sqrt{5}}{2}}{\frac{1}{8}}$$

$$f\circ g=3(2)^2(-4\sqrt{5})=-48\sqrt{5}$$

$$g(x)=(x^2-1)^{-\frac{1}{2}}\to g'(x)=-\frac{1}{2}(x^2-1)^{-\frac{3}{2}}\times 2x\to g'\left(\frac{\sqrt{5}}{2}^-\right)=\frac{1}{\sqrt{(\frac{5}{4})^--1}}=\frac{1}{\sqrt{(\frac{1}{4})^-}}=\frac{1}{(\frac{1}{2})^-}=2^+$$

$$\frac{-48\sqrt{5}}{-48\sqrt{5}}=1\quad ؟\qquad f'(2^+)=((2x)^3)'=(8x^3)'=24x^2=24\times 4$$

$$\to g'\left(\frac{\sqrt{5}}{2}^-\right)\times f'\left(g\left(\frac{\sqrt{5}}{2}\right)^-\right)=-4\sqrt{5}\times 4\times 24\to\frac{24\times 4\times(-4\sqrt{5})}{-48\sqrt{5}}=8$$

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