نام و نام خانوادگی .....آتنا ابراهیمی(?)........ ۱۸/۵ پاسخنامه تشریحی تکلیف شماره ..۲۴... کلاس .....[illegible]...

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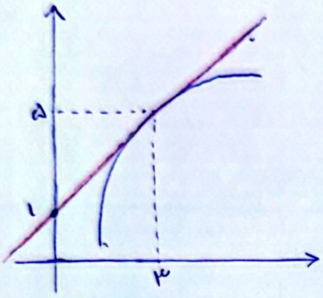

$$f'(3) = \frac{5-1}{3} = \frac{4}{3}$$

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$f(x) = \sqrt{ax-1} \to f'(x) = \dfrac{a}{2\sqrt{ax-1}}$

شیب: $\dfrac{1-2}{-1-2} = \dfrac{1}{3}$

معادله خط مماس: $y = \dfrac{x+4}{3}$

$\Rightarrow \dfrac{x+4}{3} = \sqrt{ax-1}$, $\dfrac{a}{2\sqrt{ax-1}} = \dfrac{1}{3}$

$\Rightarrow 9a = 2x + 8 \to 5a = \dfrac{10x+40}{9}$

$f(5) = \sqrt{5a-1} = \sqrt{\dfrac{50+40}{9} - \dfrac{9}{9}} = \sqrt{\dfrac{81}{9}} = \sqrt{9} = 3$

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$f(x) = \dfrac{x^2+mx+1}{x+3}$ $\quad y = \dfrac{3x+n}{4}$ $\quad f(1) = \dfrac{3+n}{4} \Rightarrow \dfrac{2+\overset{-2}{m}}{\cancel{4}} = \dfrac{3+n}{\cancel{4}}$

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$f'(x) = \dfrac{(2x+m)(x+3)-(x^2+mx+1)}{(x+3)^2} = \dfrac{x^2+6x+3m-1}{(x+3)^2}$ $\quad \boxed{n=-3}$

$\left(1, \dfrac{2+m}{4}\right) \to y' = \dfrac{(2+m)4-(2+m)}{16} = \dfrac{3(2+m)}{16} = \dfrac{3}{4} \to m = 2$

$f'(1) = \dfrac{3}{4} \Rightarrow \dfrac{\cancel{1}+6+3m-\cancel{1}}{16} = \dfrac{3}{4} \Rightarrow 6+3m = 12 \Rightarrow \boxed{m=-2}$ $\quad m+n = -5$

$y = \dfrac{3}{4}x + \dfrac{n}{4} \to \dfrac{3}{4} + \dfrac{n}{4} = \dfrac{2+m}{4} \to m-n = 1 \to n = 1$ $\quad m+n = 2+1 = 3$

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$f(x) = \dfrac{27-\sin^3 x}{9-\sin^2 x} = \dfrac{(\cancel{3-\sin x})(9+3\sin x+\sin^2 x)}{(\cancel{3-\sin x})(3+\sin x)}$ $\quad (3g-f)(x) = \dfrac{\cancel{9}}{\sin x+3} - \dfrac{\sin^2 x + 3\sin x + \cancel{9}}{3+\sin x}$

$= \dfrac{-\sin^2 x - 3\sin x}{3+\sin x} \rightsquigarrow (3g-f)'(x) = \dfrac{(-2\sin x\cos x - 3\cos x)(3+\sin x) - (\cos x)(-3\sin x - \sin^2 x)}{(3+\sin x)^2}$

$= \dfrac{-\cos x\,(\cancel{\sin^2 x + 6\sin x + 9})}{\cancel{(3+\sin x)^2}} = -\cos x$ $\quad (3g-f)'\left(\dfrac{5\pi}{4}\right) = -\cos\dfrac{5\pi}{4} = \dfrac{-1}{2}$

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ابتدا ضابطه fog را تشکیل می‌دهیم:

$fog(x) = \dfrac{-1}{\sqrt[5]{\dfrac{1}{x^5+|x^5|} + \left|\dfrac{1}{x^5+|x^5|}\right|}} = \dfrac{-1}{\sqrt[5]{\dfrac{2}{x^5+|x^5|}}}$

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$= \dfrac{-1}{\sqrt[5]{\dfrac{1}{x^5}}} = \dfrac{-1}{\dfrac{1}{x}} = -x$ $\quad (fog)'(x) = -1 \to (fog)'(\sqrt[5]{3}) = -1$

$f(x)=\left(\dfrac{-1+\sin x}{1+\sin x}\right)^2 \quad g(x)=\dfrac{f(x)-1}{x}=\dfrac{\dfrac{\sin^2 x-2\sin x+1}{\sin^2 x+2\sin x+1}-1}{x} \Rightarrow$ $g(x)=\dfrac{-4\sin x}{x(\sin^2 x+2\sin x+1)} \Rightarrow \lim_{x\to 0} g(x)=\lim_{x\to 0}\dfrac{-4\sin x}{x(\sin x+1)^2}$ ۲ $\xrightarrow{\text{هم‌ارزی}} \dfrac{-4x}{x(x+1)^2}=\dfrac{-4}{1}=-4$ ✓	۶
$y=x^2+1 \xrightarrow[\text{محور } x]{\text{قرینه}} y=-x^2-1 \quad d: y=c$ $-x^2-1=c \rightarrow x^2=-1-c \rightarrow x=\pm\sqrt{-1-c}$ $y'=-2x \quad \left.\begin{array}{l} m_1=-2\sqrt{-1-c} \\ m_2=2\sqrt{-1-c}\end{array}\right\\} m_1\times m_2=-1 \quad \cancel{-}4(-1-c)=\cancel{-}1$ ۲ $-1-c=\dfrac{1}{4} \rightarrow c=\dfrac{-5}{4}$ فاصله $=\dfrac{5}{4}$ ✓	۷
$f(x)=2\sqrt{x}(4x^2+3) \rightarrow f'(x)=\dfrac{20x^2+3}{\sqrt{x}}$, $d: y=ax$ $\Rightarrow \dfrac{20x^2+3}{\sqrt{x}}=a \rightarrow a\sqrt{x}=20x^2+3$, $ax=2\sqrt{x}(4x^2+3) \rightarrow a\sqrt{x}=8x^2+6$ $\Rightarrow 8x^2+6=20x^2+3 \rightarrow \cancel{12}\overset{4}{}x^2=\cancel{3} \rightarrow x^2=\dfrac{1}{4}$ ۲ $x=\pm\dfrac{1}{2} \xrightarrow{x>0} x=\dfrac{1}{2} \rightsquigarrow a=\dfrac{20(\frac{1}{2})^2+3}{\sqrt{\frac{1}{2}}} \Rightarrow a=8\sqrt{2}$ ✓	۸
$d: y=ax \quad f(x)=\dfrac{\sqrt{x}}{-2x^2+x+1}=ax \longrightarrow f'(x)=\dfrac{6x^2-x+1}{2\sqrt{x}(-2x^2+x+1)^2}=a$ ۱٫۵ $a\sqrt{x}=\dfrac{1}{-2x^2+x+1}$, $a\sqrt{x}=\dfrac{6x^2-x+1}{2(-2x^2+x+1)^2} \Rightarrow \dfrac{6x^2-x+1}{2(-2x^2+x+1)^2}=\dfrac{1}{-2x^2+x+1}$ $6x^2-x+1=-4x^2+2x+2 \rightarrow 10x^2-3x-1=0$ $x=\dfrac{1}{2}$, $x=-\dfrac{1}{5}$ غ ق ق $f(\alpha)=\dfrac{\sqrt{\frac{1}{2}}}{-2(\frac{1}{2})^2+\frac{1}{2}+1}=\dfrac{\sqrt{2}}{2}$	۹
$f(x)=(x[x])^3 \rightarrow f'(x)=3(x[x])^2\times[x]=3[x](x[x])^2$ $g(\frac{\sqrt{5}}{2})=\dfrac{1}{\sqrt{(\frac{\sqrt{5}}{2})^2-1}}=2 \longrightarrow f(g(\frac{\sqrt{5}}{2}))=f(2)=96$ $g'(\frac{\sqrt{5}}{2})=\dfrac{-\frac{\sqrt{5}}{2}}{((\frac{\sqrt{5}}{2})^2-1)^{\frac{3}{2}}}=1/8 \rightarrow g'(\frac{\sqrt{5}}{2})=\dfrac{-\frac{\sqrt{5}}{2}}{1/8}=-4\sqrt{5}$ ۲ $(f(g(x)))'=g'(x)f'(g(x)) = 96\times -4\sqrt{5} = -384\sqrt{5}$ $\dfrac{-384\sqrt{5}}{-48\sqrt{5}}=8$ ✓	۱۰