دوازدهم ریاضی ۹ — شنبه ۱۴ — نگین نکوئی — 14,5

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$(0,1) \quad (3,5) \longrightarrow m = \frac{\Delta y}{\Delta x} = \frac{5-1}{3-0} = \frac{4}{3}$

$f'(3) = \frac{4}{3}$ ✓

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$(2,2)\ (-1,1) \longrightarrow m = \frac{\Delta y}{\Delta x} = \frac{2-1}{2-(-1)} = \frac{1}{3}$

$y = mx + b \Rightarrow y = \frac{1}{3}x + \frac{4}{3}$

$f(x) = \sqrt{ax-1} \Rightarrow 3\sqrt{ax-1} = x+4$

$9(ax-1) = x^2 + 16 + 8x$

$9ax - 9 = x^2 + 8x + 16 \rightarrow x^2 + (8-9a)x + 25 = 0$

۲

$8 - 9a = 10 \rightarrow -2 = 9a \quad a = -\frac{2}{9}$

$8 - 9a = -10 \rightarrow 18 = 9a \quad a = 2$ قبول

$x = 5 \rightarrow \sqrt{2a-1} = \sqrt{9} = 3$ ✓

۳ -

$4y - 3x = n$

$\rightarrow m = \dfrac{-\text{ضریب } x}{\text{ضریب } y} = \dfrac{-(-3)}{4} = \dfrac{3}{4}$

$\longrightarrow y = \dfrac{x^2 + nx + 1}{x+3} \Rightarrow y' = \dfrac{(2x+n)(x+3) - x^2 - nx - 1}{(x+3)^2}$

$= \dfrac{2x^2 + (6+n)x + 3n - x^2 - nx - 1}{x^2 + 6x + 9} \xrightarrow{x=1} \dfrac{6 + 3n - n - 1}{16}$

$= \dfrac{x^2 + 6x + 3n - 1}{16} = \dfrac{6 + 3n}{16} = \dfrac{3}{4}\;\; \dfrac{12}{6}$

$\rightarrow \dfrac{6 + 3n}{16} = \dfrac{12}{1} \rightarrow \cancel{\;}$

$\boxed{n = 2}$

$\rightarrow x = 1$

$y = \dfrac{1 + 2(1) + 1}{4} = 1$

$4y - 3x = n \rightarrow 4 - 3 = 1 \rightarrow n = 1$

$m + n = 3$

(۲)

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$f(x) = \dfrac{(3 - \cancel{\sin x})(9 + \sin^2 x + 3\sin x)}{(3 - \cancel{\sin x})(3 + \sin x)}$

$\rightsquigarrow f'(x) = \dfrac{(3\sin x \cos x + 3\cos x)(3 + \sin x) - (\cos x)(9 + \sin^2 x + 3\sin x)}{(9 - \sin^2 x)^2}$

$\sin \dfrac{2\pi}{3} = -\dfrac{\sqrt{3}}{2}$

$\cos \dfrac{2\pi}{3} = \dfrac{1}{2}$

(۲)

$g(x) = \dfrac{3}{3 + \sin x}$

$= (3g - f)x = \dfrac{9 - (9 + \sin^2 x + 3\sin x)}{3 + \sin x} = \dfrac{-\sin x(\sin x + 3)}{\sin x + 3} = -\sin x$

$\xrightarrow{\text{مشتق}} -\cos x \xrightarrow{x = \frac{2\pi}{3}} -\cos \dfrac{2\pi}{3} = -\left(\dfrac{1}{2}\right) = -\dfrac{1}{2}$

۵ –

$g'(\sqrt[3]{2})\, f'(g(\sqrt[3]{2}))$

$(f\circ g)' = f(g(x)) = \dfrac{1}{\sqrt[5]{\frac{1}{2x^5}+\frac{1}{2x^5}}\;x^{-1}} = -x \xrightarrow[\text{منفی}]{f'} (-1)$

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۶ –

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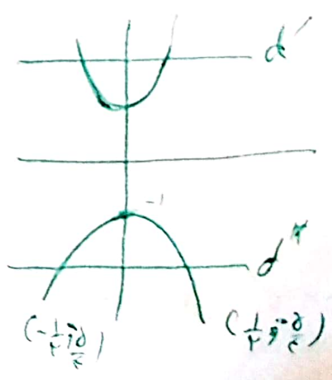
$f(x) = xg(x)+1 \longrightarrow g(x)=\dfrac{f(x)-1}{x}$

$\cancel{f'(x) = g(x) + g'(x)\,x}$

$g(x) = \dfrac{\left(\frac{\sin x-1}{\sin x+1}\right)^2-1}{x} \xrightarrow{x\to 0}$ صورت $\dfrac{\left(\frac{x-1}{x+1}\right)^2-1}{x}$

$= \dfrac{x^2-2x+1-x^2-2x-1}{x(x^2+2x+1)} = \dfrac{-4x}{x(x^2+2x+1)} \Rightarrow \lim_{x\to 0} g(x) = \dfrac{-4}{1} = -4$

۷ –




$y=x^2+1 \longrightarrow f'(x)=2x$

$2\alpha \times 2\beta = -1$

$\alpha\beta = -\frac{1}{4} \longrightarrow \alpha=-\frac{1}{2},\ \beta=\frac{1}{2}$

$\rightarrow -\frac{a}{4}\cdot\circ = \frac{a}{4}$ عرض از مبدا

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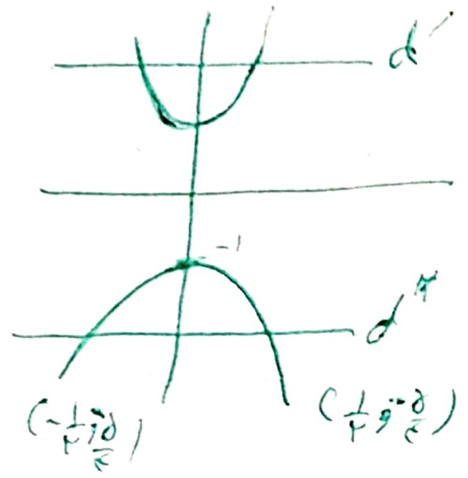
7 -




$g = x^2 + 1 \longrightarrow f'(x) = 2x$

$2\alpha \times 2\beta = -1$

$\alpha\beta = -\frac{1}{4} \longrightarrow \alpha = -\frac{1}{2}$, $\beta = \frac{1}{2}$

$\longrightarrow -\frac{5}{4} - 0 = \frac{5}{4}$ فاصله خط از مبدأ

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8 - 1,5

$f(x) = 2\sqrt{x}\,(4x^2+3) \longrightarrow f'(x) = \frac{1}{\sqrt{x}}(4x^2+3) + (8x)(2\sqrt{x})$

$= \frac{4x^2+3+16x^2}{\sqrt{x}} = \frac{20x^2+3}{\sqrt{x}} = 16x\sqrt{x} + \frac{3}{\sqrt{x}} = 20a\sqrt{a} + \frac{3}{\sqrt{a}} \xrightarrow{=m \times a} 20a^2\sqrt{a} + 3\sqrt{a} = ma$

$d = mx$ $\quad (a, ma)$

$f(a) = 8a^2\sqrt{a} + 6\sqrt{a} \rightarrow 12a^2\sqrt{a} = 3\sqrt{a} \xrightarrow{\div 3\sqrt{a}} 4a^2 = 1$

$a = \pm\frac{1}{2}$

$a = \frac{1}{2}$ قابل قبول

$a = -\frac{1}{2}$ غیرقابل قبول چون در دامنه نیست

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$\rightarrow 2(4\alpha^2+3) = 20\alpha^2 + 3 \rightarrow 12\alpha^2 = 3 \rightarrow \alpha^2 = \frac{1}{4}$

$m = \frac{20(\frac{1}{4})+3}{\sqrt{\frac{1}{4}}} = 8\sqrt{4}$

۹ -

$d: y = cx$

$$f(a) = \frac{\sqrt{a}}{-2a^2 + a + 1} = c\sqrt{a}$$

$$c\sqrt{a}(-2a^2 + a + 1) = 1 \longrightarrow -2ca^{\frac{5}{2}} + ca^{\frac{3}{2}} + ca^{\frac{1}{2}} = 1$$

$$\xrightarrow[f'(x)]{\text{مشتق}} -5ca^{\frac{3}{2}} + \frac{3}{2}ca^{\frac{1}{2}} + \frac{1}{2}ca^{-\frac{1}{2}} = 0 \rightarrow -5a\sqrt{a} + \frac{3}{2}\sqrt{a} + \frac{1}{2\sqrt{a}} = 0$$

$$-10a^2 + 3a + 1 = 0 \rightarrow a = -\frac{1}{5},\ a = \frac{1}{2}$$

$$f(a) = \frac{\sqrt{\frac{1}{2}}}{-2(\frac{1}{4}) + \frac{1}{2} + 1} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$f(x) = (x[x])^3$ — ۱۰

$\rightarrow g(x) = \frac{1}{\sqrt{x^2-1}} \Rightarrow (f \circ g)' = g'(\frac{\sqrt{5}}{2}) \cdot f'(g(\frac{\sqrt{5}}{2}))$

$[x] \xrightarrow{x=2^+} [x] = 2$

$f(x) = (2x)^3 = 8x^3 \Rightarrow f'(x) = 24x^2 = 24 \times 4$

$\rightsquigarrow \frac{24 \times 4 \times 4\sqrt{5}}{\cancel{4} \times \cancel{\sqrt{5}}} = 24 \times 4 = 96$ ②

$g'(\frac{\sqrt{5}}{2}) = \frac{0 - \frac{2x}{2\sqrt{x^2-1}}}{x^2-1} = \frac{\frac{\sqrt{5}}{\frac{1}{2}}}{\frac{1}{4}} = 4\sqrt{5}$