روشنا ایوبی

۱۹٫۸

(۱) $A(3,5)\ ,\ B(0,1) \Rightarrow m = \frac{5-1}{3-0} = \frac{4}{3} \Rightarrow f'(3) = \boxed{\frac{4}{3}}$ ✓ (۲)

(۲) $(2,2)\ ,\ (-1,1) \Rightarrow m = \frac{2-1}{2-(-1)} = \frac{1}{3} \Rightarrow y = \frac{x}{3} + \frac{4}{3}$

$\Rightarrow \frac{1}{3}x + \frac{4}{3} = \sqrt{ax-1} \Rightarrow x+4 = 3\sqrt{ax-1} \Rightarrow x^2+8x+16 = 9ax-9$

$\Rightarrow x^2+(8-9a)x+25=0 \Rightarrow \Delta=0 \Rightarrow (8-9a)^2-4(25)=0$ (۲)

$\Rightarrow \underset{ق}{a=2}\ ,\ a=-\frac{2}{9} \longrightarrow f(5)=\sqrt{\frac{-19}{9}}$ × غ ق ق ✓

$f(5)=\sqrt{9}=\boxed{3}$ ✓

(۳) $4y-3x=n \Rightarrow m=\frac{3}{4}$ ، $f'(1)=\frac{(2x+m)(4)-(2+m)}{16}$

$=\frac{3(2+m)}{16}=\frac{3}{4} \Rightarrow m=2$ ، $x=1 \Rightarrow y=1 \Rightarrow 4-3=1 \Rightarrow n=1$ ✓ (۲)

$\Rightarrow m+n=2+1=\boxed{3}$

(۴) $(3g-f)'(\frac{5\pi}{3}) = \frac{9}{3+\sin x} - \frac{(3-\sin x)(9+3\sin x+\sin^2 x)}{(3-\sin x)(3+\sin x)}$

$=\frac{9-9-3\sin x-\sin^2 x}{(3+\sin x)} = \frac{-\sin x(3+\sin x)}{(3+\sin x)} = (-\sin x)' = -\cos x$

$= -\cos\frac{5\pi}{3} = \boxed{-\frac{1}{2}}$ ✓ (۲)

(۵) $g'(\sqrt[5]{3})\, f'(g(\sqrt[5]{3})) = f(g(\sqrt[5]{3}))'$ (۲)

$x>0 \Rightarrow fog(x) = \frac{1}{\sqrt[5]{\frac{1}{2x^5}+\frac{1}{2x^5}}} = \frac{-1}{\sqrt[5]{\frac{2}{2x^5}}} = -\frac{1}{\frac{1}{x}} = -x \xrightarrow{\text{مشتق}} \boxed{-1}$ ✓

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روشنا الوسی

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$$g(x)=\frac{f(x)-1}{x}\xrightarrow{\text{هم‌ارزی}}\frac{\left(\frac{1+x}{1-x}\right)^2-1}{x}=\frac{\frac{x^2+2x+1-x^2+2x-1}{x^2-2x+1}}{x}$$

$f'(0)=2\times\left(\frac{2}{1}\right)\times(-1)=-4$

$$=\frac{4x}{x(x^2+2x+1)}=\frac{4}{(x+1)^2}\xrightarrow{x=0}=\boxed{-4}$$

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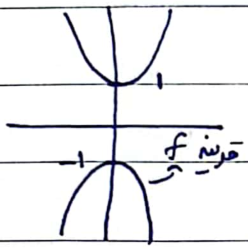

$\to y=k \Rightarrow k<-1$

$-y=x^2+1 \Rightarrow x^2=-k-1 \Rightarrow x=\pm\sqrt{-k-1}$

$$y'=-2x \to \begin{cases} x=\sqrt{-k-1} \Rightarrow y'=-2\sqrt{-k-1} \\ x=-\sqrt{-k-1} \Rightarrow y'=2\sqrt{-k-1} \end{cases}$$

$-4(-k-1)=-1 \xleftarrow{\text{عمود}}$

$\Rightarrow k=-\frac{5}{4} \xrightarrow{\text{فاصله از مبدأ}} \boxed{k=\frac{5}{4}}$

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$2\sqrt{x}\,(4x^2+3)=ax \Rightarrow a=\frac{2\sqrt{x}\,(4x^2+3)}{x}$

$\frac{2}{2\sqrt{x}}(4x^2+3)+8x(2\sqrt{x})=a \Rightarrow 4x^2+3+16x^2=a\sqrt{x}$

$\Rightarrow 20x^2+3=\frac{2x(4x^2+3)}{x} \Rightarrow 20x^2+3=8x^2+6$

$\Rightarrow 12x^2=3 \Rightarrow x^2=\frac{3}{12} \Rightarrow x=\pm\frac{1}{2} \Rightarrow x=\frac{1}{2}$ $\quad m=\frac{20\left(\frac{1}{4}\right)+3}{\sqrt{\frac{1}{2}}}=8\sqrt{2}$

$\Rightarrow 2\sqrt{\frac{1}{2}}\times 4=2a \Rightarrow a=\frac{4}{\sqrt{2}}=\boxed{2\sqrt{2}} \to$ فاصله‌ی خط d

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$$fog(x)=\left(\frac{1}{\sqrt{x^2-1}}\left[\frac{1}{x^2-1}\right]\right)^2 \Rightarrow fog(x)=\left(\frac{2}{\sqrt{x^2-1}}\right)^2=\frac{4}{x^2-1}$$

$x<\frac{\sqrt{5}}{2} \Rightarrow x^2<\frac{5}{4} \Rightarrow x^2-1<\frac{1}{4} \Rightarrow \sqrt{x^2-1}<\frac{1}{2} \Rightarrow \frac{1}{\sqrt{x^2-1}}>2$

$\Rightarrow \left[\frac{1}{\sqrt{x^2-1}}\right]=2$ $\quad \left(fog\left(\frac{\sqrt{5}}{2}\right)\right)'=g'\left(\frac{\sqrt{5}}{2}\right)^-\times f'\left(g\left(\frac{\sqrt{5}}{2}\right)^-\right)$

$$\frac{-64\sqrt{5}}{-48\sqrt{5}}=\boxed{\frac{4}{3}}$$

$$(fog)'(x)=\frac{-8x}{(x^2-1)^2}\xrightarrow{x=\frac{\sqrt{5}}{2}}\frac{-8\left(\frac{\sqrt{5}}{2}\right)}{\left(\frac{5}{4}-1\right)^2}=-64\sqrt{5}$$

$g(x)=(x^2-1)^{-\frac{1}{2}} \to g'(x)=-\frac{1}{2}(x^2-1)^{-\frac{3}{2}}\times 2x \to g'\left(\frac{\sqrt{5}}{2}^-\right)=\frac{1}{\sqrt{\left(\frac{5}{4}\right)^- -1}}=\frac{1}{\sqrt{\left(\frac{1}{4}\right)^-}}=\frac{1}{\left(\frac{1}{2}\right)^-}=2^+$

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$f'(2^+)=((2x)^3)'=(8x^3)'=24x^2=24\times 4 \to g'\left(\frac{\sqrt{5}}{2}^-\right)\times f'\left(g\left(\frac{\sqrt{5}}{2}\right)^-\right)=-4\sqrt{5}\times 4\times 24 \to \frac{24\times 4\times(-4\sqrt{5})}{-48\sqrt{5}}=8$

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سوال ۹

d : خط $\rightarrow y = ax$ $\qquad A(\alpha, a\alpha)$

$$f(\alpha) = \frac{\sqrt{\alpha}}{-2\alpha^2 + \alpha + 1} = a\alpha \longrightarrow a\sqrt{\alpha}\,(-2\alpha^2 + \alpha + 1) = 1 \rightarrow -2a\alpha^{\frac{5}{2}} + a\alpha^{\frac{1}{2}} + a\alpha^{\frac{3}{2}} = 1$$

$$\xrightarrow{\text{مشتق}} -5a\alpha^{\frac{3}{2}} + \frac{3}{2}a\alpha^{\frac{1}{2}} + \frac{1}{2}a\alpha^{-\frac{1}{2}} = 0 \xrightarrow[\times 2\sqrt{\alpha}]{\div a} -10\alpha^2 + 3\alpha + 1 = 0 \longrightarrow \begin{cases} \alpha = -\frac{1}{5} \\ \alpha = \frac{1}{2} \end{cases}$$

$$f(\alpha) = \frac{\sqrt{\frac{1}{2}}}{-2\left(\frac{1}{2}\right)^2 + \frac{1}{2} + 1} = \frac{\sqrt{2}}{2}$$