روشا ایوبی

۱) $A(۳,۵)\ ,\ B(۰,۱) \Rightarrow m = \frac{۵-۱}{۳-۰} = \frac{۴}{۳} \Rightarrow f'(۳) = \boxed{\frac{۴}{۳}}$

۲) $(۲,۲)\ ,\ (-۱,۱) \Rightarrow m = \frac{۲-۱}{۲-(-۱)} = \frac{۱}{۳} \Rightarrow y = \frac{x}{۳} + \frac{۴}{۳}$

$\Rightarrow \frac{۱}{۳}x + \frac{۴}{۳} = \sqrt{ax-۱} \Rightarrow x+۴ = ۳\sqrt{ax-۱} \Rightarrow x^۲+۸x+۱۶ = ۹ax-۹$

$\Rightarrow x^۲+(۸-۹a)x+۲۵=۰ \Rightarrow \Delta=۰ \Rightarrow (۸-۹a)^۲-۴(۲۵)=۰$

$\Rightarrow a=۲\ ,\ a=-\frac{۲}{۹} \longrightarrow f(۵)=\sqrt{\frac{-۱۹}{۹}}$ × غقق

$f(۵)=\sqrt{۹}=\boxed{۳}$ ✓

۳) $۴y-۳x=n \Rightarrow m=\frac{۳}{۴}$; $f'(۱)=\frac{(۲x+m)(۴)-(۲+m)}{۱۶}$

$= \frac{۳(۲+m)}{۱۶} = \frac{۳}{۴} \Rightarrow m=۲$, $x=۱ \Rightarrow y=۱ \Rightarrow ۴-۳=۱ \Rightarrow n=۱$

$\Rightarrow m+n=۲+۱=\boxed{۳}$

۴) $(۳g-f)'(\frac{۵\pi}{۳}) = \frac{۹}{۳+\sin x} - \frac{(۳-\sin x)(۹+۳\sin x+\sin^۲ x)}{(۳-\sin x)(۳+\sin x)}$

$= \frac{۹-۹-۳\sin x-\sin^۲ x}{(۳+\sin x)} = \frac{-\sin x(۳+\sin x)}{(۳+\sin x)} = (-\sin x)' = -\cos x$

$= -\cos\frac{۵\pi}{۳} = \boxed{-\frac{۱}{۲}}$

۵) $g'(\sqrt[۳]{۳})\,f'(g(\sqrt[۳]{۳})) = f(g(\sqrt[۳]{۳}))'$

$x>۰ \Rightarrow fog(x) = -\frac{۱}{\sqrt[۵]{\frac{۱}{۲x^۵}+\frac{۱}{۲x^۵}}} = \frac{-۱}{\sqrt[۵]{\frac{۲}{۲x^۵}}} = -\frac{۱}{\frac{۱}{x}} = -x \xrightarrow{\text{مشتق}} \boxed{-۱}$

رونیا الوسی

۶

$$g(x)=\frac{f(x)-1}{x}\xrightarrow{\text{هم‌ارزی}}\frac{\left(\frac{1+x}{1-x}\right)^2-1}{x}=\frac{\frac{x^2+2x+1-x^2+2x-1}{x^2-2x+1}}{x}$$

$$=\frac{4x}{x(x^2+2x+1)}=\frac{4}{(x+1)^2}\xrightarrow{x=0}=\boxed{4}$$

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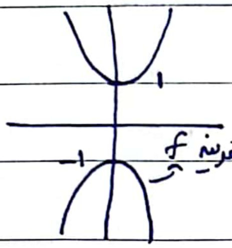

$\to y=k \Rightarrow k<-1$

$-y=x^2+1 \Rightarrow x^2=-k-1 \Rightarrow x=\pm\sqrt{-k-1}$

$$y'=-2x \to \begin{cases} x=\sqrt{-k-1} \Rightarrow y'=-2\sqrt{-k-1} \\ x=-\sqrt{-k-1} \Rightarrow y'=2\sqrt{-k-1} \end{cases}$$

$-4(-k-1)=-1 \xleftarrow{\text{عمود}}$

$$\Rightarrow k=-\frac{5}{4}\xrightarrow{\text{فاصله از مبدأ}}\boxed{k=\frac{5}{4}}$$

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$$2\sqrt{x}\,(4x^2+3)=ax \Rightarrow a=\frac{2\sqrt{x}\,(4x^2+3)}{x}$$

$$\frac{2}{2\sqrt{x}}(4x^2+3)+8x(2\sqrt{x})=a \Rightarrow 4x^2+3+16x^2=a\sqrt{x}$$

$$\Rightarrow 20x^2+3=\frac{2x(4x^2+3)}{x} \Rightarrow 20x^2+3=8x^2+6$$

$$\Rightarrow 12x^2=3 \Rightarrow x^2=\frac{3}{12} \Rightarrow x=\pm\frac{1}{2} \Rightarrow x=\frac{1}{2}$$

$$\Rightarrow 2\sqrt{\tfrac{1}{2}}\times 4=2a \Rightarrow a=\frac{4}{\sqrt{2}}=\boxed{2\sqrt{2}}$$ → فاصله‌ی خط تا

۱۰

$$fog(x)=\left(\frac{1}{\sqrt{x^2-1}}\left[\frac{1}{x^2-1}\right]\right)^2 \Rightarrow fog(x)=\left(\frac{2}{\sqrt{x^2-1}}\right)^2=\frac{4}{x^2-1}$$

$$x<\frac{\sqrt{5}}{2} \Rightarrow x^2<\frac{5}{4} \Rightarrow x^2-1<\frac{1}{4} \Rightarrow \sqrt{x^2-1}<\frac{1}{2} \Rightarrow \frac{1}{\sqrt{x^2-1}}>2$$

$$\Rightarrow \left[\frac{1}{\sqrt{x^2-1}}\right]=2$$

$$(fog)'(x)=\frac{-8x}{(x^2-1)^2}\xrightarrow{x=\frac{\sqrt{5}}{2}}\frac{-8\left(\frac{\sqrt{5}}{2}\right)}{\left(\frac{5}{4}-1\right)^2}=-64\sqrt{5}$$

$$\frac{-64\sqrt{5}}{-48\sqrt{5}}=\boxed{\frac{4}{3}}$$