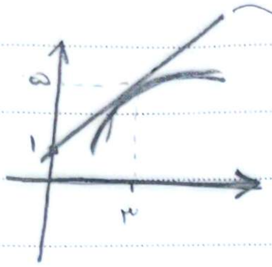
دوازدهم ریاضی A — ... ۱۸٫۵

Subject: تکلیف 24 مشتق
Year: Month: Day:

(۱)



$y = ax + b$

$\to f'(3) = a \to \frac{\Delta y}{\Delta x}$

دو نقطه: $(0,1)$ و $(3,5)$

$\frac{\Delta y}{\Delta x} \Rightarrow \frac{5-1}{3-0} = \frac{4}{3}$ $\qquad f'(3) = \frac{4}{3}$

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$f(x) = \sqrt{ax-1} \xrightarrow{\text{مشتق}} f'(x) = \frac{a}{2\sqrt{ax-1}}$

شیب خط $\Rightarrow \frac{\Delta y}{\Delta x} \to \frac{2-1}{2-(-1)} = \frac{1}{3}$ $\qquad (2,2) \to \frac{1}{3}x + b = y \to \frac{2}{3} + b = \frac{6}{3}$

$b = \frac{4}{3}$

$f'(A) = \frac{a}{2\sqrt{aA-1}} = \frac{1}{3} \to 3a = 2\sqrt{aA-1}$ ★

$\to a > 0$

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$\sqrt{aA-1} = \frac{A}{3} + \frac{4}{3} \to \frac{3}{2}a = \frac{A}{3} + \frac{4}{3} \to \frac{9}{2}a - 4 = A$ ★★

$\xrightarrow{★} 9a^2 = 4(aA-1) \xrightarrow{★★} 9a^2 = 4a\left(\frac{9}{2}a - 4\right) - 4 \to 9a^2 = 18a^2 - 16a - 4$

$0 = 9a^2 - 16a - 4$

$f(5) = \sqrt{2x-1} \to \sqrt{10-1} = 3$ $\qquad 2 = a \leftarrow \frac{16+20}{18} \xleftarrow{a>0} \frac{16 \pm 4(5)}{18} \leftarrow a = \frac{16 \pm \sqrt{16(16+9)}}{18}$

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$y = \frac{x^2 + mx + 1}{x + 3}$ $\qquad 4y - 3x = n \xrightarrow{\text{شیب خط}} \frac{3}{4}$

$4y = 3x + n$

$y = \frac{3}{4}x + \frac{n}{4}$

$y' = \frac{(2x+m)(x+3) - x^2 - mx - 1}{(x+3)^2} \xrightarrow{x=1} \frac{4(2+m) - 2 - m}{16} = \frac{3}{4}$

$\to \frac{8 + 4m - 2 - m}{16} = \frac{3}{4} \to \frac{3m+6}{16} = \frac{3}{4} \to 12 = 3m + 6$

$\boxed{m=2}$

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$y = \frac{x^2 + 2x + 1}{x + 3} \xrightarrow{x=1} y = \frac{1+2+1}{4} = 1$

$(1,1) \to y = \frac{3}{4}x + \frac{n}{4} \to 1 = \frac{3}{4} + \frac{n}{4}$

$\boxed{n=1}$

$m + n = 2 + 1 = 3$

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$$f(x) = \frac{27 - \sin^3 x}{9 - \sin^2 x} \xrightarrow{\text{ساده کردن}} \frac{(3-\sin x)(9+\sin^2 x + 3\sin x)}{(3-\sin x)(3+\sin x)} = \frac{\sin^2 x + 3\sin x + 9}{3+\sin x}$$

صورت: تفاضل مکعب
مخرج: مزدوج

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$$g(x) = \frac{3}{3+\sin x}$$

$$(3g - f)(x) = \frac{9 - (9 + \sin^2 x + 3\sin x)}{3 + \sin x} = \frac{-\sin x(\sin x + 3)}{\sin x + 3} = -\sin x$$

$$(3g-f)'(x) \longrightarrow -\cos x \xrightarrow{x=\frac{5\pi}{3}} -\cos\frac{5\pi}{3} \longrightarrow \frac{-1}{2}$$

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$$f(x) = \frac{-1}{\sqrt[5]{x+|x|}} \qquad g(x) = \frac{1}{x^5+|x|^5} \xrightarrow{x=\sqrt[5]{3}} g(x) = \frac{1}{2x^5}$$

$$x = \sqrt[5]{3} \longrightarrow f(x) = \frac{-1}{\sqrt[5]{2x}}$$

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$$(f\circ g)(x) = \frac{1}{2\left(\frac{-1}{\sqrt[5]{2x}}\right)^5} = \frac{1}{2\left(\frac{-1}{2x}\right)} = -x \longrightarrow (f\circ g)'(x) = -1$$

$$(f\circ g)'(\sqrt[5]{3}) = -1$$

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$$f(x) = \left(\frac{-1+\sin x}{1+\sin x}\right)^2 \qquad f(x) = x\,g(x) + 1$$

$$g(x) = \frac{f(x)-1}{x} \qquad \lim_{x\to 0} g(x)$$

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$$\to g(x) = \left(\frac{\sin x - 1}{\sin x + 1}\right)^2 - 1 \xrightarrow{x\to 0} \text{هم‌ارزی} \ \frac{\left(\frac{x-1}{x+1}\right)^2 - 1}{x} \to \frac{x^2 - 2x + 1 - x^2 - 2x - 1}{x(x^2+2x+1)}$$

$$\to \frac{-4x}{x(x^2+2x+1)} = -4$$

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$$y = x^2 + 1 \xrightarrow{\text{قرینه نسبت به } x} y = -x^2 - 1 \to k = -x^2 - 1$$

$$y = k$$

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$$y' = -2x \to x = +\sqrt{-k-1}$$

$$\hookrightarrow x = -\sqrt{-k-1} \Rightarrow y' = +2\sqrt{-k-1}$$

$$\xrightarrow{mm' = -1} -4(-k-1) = -1$$

$$-k-1 = \frac{1}{4} \to k = \frac{-5}{4}$$

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$$y = \frac{-5}{4} \xrightarrow[(op)]{\text{فاصله}} \frac{5}{4}$$

$f(x) = 12\sqrt{x}(4x^2+3) = 48x^2\sqrt{x} + 6\sqrt{x} \longrightarrow f'(x) = 120x\sqrt{x} + \frac{3}{\sqrt{x}} = \frac{120x^2+3}{\sqrt{x}}$

$y - 12\sqrt{\alpha}(4\alpha^2+3) = \frac{120\alpha^2+3}{\sqrt{\alpha}}(x-\alpha) \xrightarrow{(0,0)} -12\sqrt{\alpha}(4\alpha^2+3) = \frac{120\alpha^2+3}{\sqrt{\alpha}}(-\alpha)$

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8) ماگزیمم ____ تقاطع $(0,0)$ ←—— شیب خط $= f'(\alpha)$

$f'(x) = \frac{1}{\sqrt{x}}(4x^2+3) + 2\sqrt{x}(4x)$ تانژانت $+0$

$\to 2(4\alpha^2+3) = 120\alpha^2+3 \to 112\alpha^2 = 3 \to \alpha^2 = \frac{1}{4}$ موازی بودن‌ها

تن شیب خط:

$m = \frac{120(\frac{1}{4})+3}{\sqrt{\frac{1}{4}}} = 18\sqrt{2}$

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9) $f(x) = \frac{\sqrt{x}}{-2x^2+x+1}$

$\to Kx = \frac{\sqrt{x}}{-2x^2+x+1} \longrightarrow K = \frac{1}{\sqrt{x}(-2x^2+x+1)}$

خط $\Rightarrow y = Kx$

شیب $\longrightarrow y' = K$

$\hookrightarrow f'(x) = \frac{\frac{1}{2\sqrt{x}}(-2x^2+x+1) - (-4x+1)(\sqrt{x})}{(-2x^2+x+1)^2} = \frac{5x^2-x+1}{2\sqrt{x}(-2x^2+x+1)^2}$

$K = \frac{5x^2-x+1}{2\sqrt{x}(-2x^2+x+1)^2} = \frac{1}{\sqrt{x}(-2x^2+x+1)} \longrightarrow -4x^2+2x+2-5x^2+x-1=0$

$10x^2-3x-1=0 \longrightarrow x=\frac{1}{2}$

$\hookrightarrow x=-\frac{1}{5}$ در x زیر رادیکال بوده غ‌ق‌ق

$f(\frac{1}{2}) \to \frac{\sqrt{2}}{2}$

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10) $f(x) = (x[x])^3$ $g(x) = \frac{1}{\sqrt{x^2-1}}$ $x = \frac{\sqrt{5}}{2}$

$\hookrightarrow g(x) = (x^2-1)^{-\frac{1}{2}}$

$g'(x) \times f'(g(x))$

$-4\sqrt{5} \times \frac{f'(2)}{96} = -384\sqrt{5}$ $g'(x) = -\frac{1}{2}(2x)(x^2-1)^{-\frac{3}{2}}$

$f(x) \Rightarrow x=2^+ \to (2x)^3 \to f'(x) = 24x^2$

$g(\frac{\sqrt{5}}{2}) = \frac{1}{\sqrt{\frac{\frac{5}{4}-\frac{4}{4}}{\frac{1}{2}}}} \longrightarrow 2$ ۲

$g'(\frac{\sqrt{5}}{2}) = (\frac{-\sqrt{5}}{2})(\frac{1}{4})^{\cancel{-\frac{3}{2}}} \longrightarrow 4\sqrt{5} \to 8$

$\frac{-384\sqrt{5}}{-48\sqrt{5}} = 8$

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