تکلیف شماره ۲۴: بسمه تعالی خدا — یکتا دستانی / دوازدهم دبیرستان A

مماس خط $\Rightarrow ax+b \xrightarrow{b=1} ax+1$ ، $f'(2)=a$ ①

$x=2 \rightarrow 2a+1 = f(2) = a \rightarrow a = 4 \rightarrow \boxed{a=\frac{4}{4}} = f'(c)$

$f(x)=\sqrt{ax-1} \rightarrow f'(x)=\frac{a}{2\sqrt{ax-1}}$ ②

شیب خط $= m = \frac{\Delta y}{\Delta x} = \frac{2-1}{2-(-1)} = \frac{1}{3}$

معادله خط $\rightarrow (y-1) = \frac{1}{3}(x+1) \rightarrow y-1 = \frac{1}{3}x + \frac{1}{3} \rightarrow y = \frac{1}{3}x + \frac{4}{3}$

$f'(x) = \frac{1}{3}x + \frac{4}{3} = \sqrt{ax-1} \rightarrow ax-1 = \frac{1}{9}x^2 + \frac{14}{9} + \frac{5}{9}x$

$\rightarrow \frac{1}{9}x^2 + (\frac{5}{9}-a)x + \frac{23}{9} = 0 \xrightarrow{\times 9} x^2 + (5-9a)x + 23 = 0$

$\Delta = 0 \rightarrow (5-9a)^2 = 100 \rightarrow \begin{cases} 25-9a = +10 \rightarrow -9a = 2 \rightarrow a = -\frac{2}{9} \\ 25-9a = -10 \rightarrow -9a = -15 \rightarrow a = 5 \end{cases}$

$\rightarrow \begin{cases} a = -\frac{2}{9} \rightarrow x = -9 \rightarrow \text{قق} \\ a = +5 \rightarrow x = +9 \rightarrow \text{ق} \end{cases} \rightarrow f(9) = \sqrt{2\times 9 - 1} = \boxed{4} \rightarrow$ گزینه ۲

Senobar

SUBJECT

Year: Month: Day:

Page: ( )

$4y - 3x = n \rightarrow y = \frac{3}{4}x + \frac{n}{4} \rightarrow$ شیب $= \frac{3}{4}$ (۳)

$\hookrightarrow x=1 \rightarrow 4y = 3+n \rightarrow y = \frac{3+n}{4}$ ؟ $\rightarrow \frac{3+n}{4} = \frac{m+2}{4}$

$\hookrightarrow x=1 \rightarrow y = \frac{1+m+1}{4} \Rightarrow y = \frac{m+2}{4}$

$\rightarrow \boxed{m-n=1}$

$y' = \frac{(2x+m)(x+2) - (x^2+mx+1)}{(x+2)^2} = \frac{2x^2+(4+m)x+2m-x^2-mx-1}{(x+2)^2}$

$\rightarrow \frac{x^2+4x+(2m-1)}{(x+2)^2} \xrightarrow{x=1} \frac{1+4+2m-1}{16} = \frac{3}{4}$

$\rightarrow \frac{4+2m}{16} = \frac{3}{4} \rightarrow 4+2m = 12+12m \rightarrow 24 = 12m \rightarrow \boxed{m=2}$

$2-n=1 \rightarrow \boxed{n=1} \Rightarrow m+n = 2+1 = \boxed{3} \rightarrow ؟.$

$2g'\left(\frac{\alpha x}{2}\right) - f'\left(\frac{\alpha x}{2}\right) = (2g-f)'\left(\frac{\alpha x}{2}\right)$ (۴)

$\frac{2\times 2}{2+\sin x} - \frac{7-\sin^2 x}{2-\sin x} = \frac{9}{2+\sin x} - \frac{(2-\sin x)(9+\sin^2 x+2\sin x)}{(2-\sin x)(2+\sin x)}$

$\rightarrow (2g-f)(x) = \frac{(9-9-\sin^2 x+2\sin x)}{2+\sin x} = \frac{-\sin x(2+\sin x)}{2+\sin x}$

$-\sin x$

$(2g-f)'(x) = -\cos x \xrightarrow{x=\frac{\alpha x}{2}} -1\times -\frac{1}{2} = \frac{1}{2} \rightarrow ؟.$

$x>0 \rightarrow f(x) = \frac{-1}{\sqrt[5]{2x}} \; , \; g(x) = \frac{1}{2x^5}$ (۵)

$g'(x) \times f'(g(x)) = (fog)'(x)$

$(fog)(x) = \frac{-1}{\sqrt[5]{\frac{2\times 1}{2x^5}}} = -\frac{1}{\frac{1}{x}} = -x \rightarrow (fog)'(x) = -1$

؟

Senobar

$$f(x)=\left(\frac{\sin x-1}{\sin x+1}\right)^2=\frac{\sin^2x-2\sin x+1}{\sin^2x+2\sin x+1}=x\,g(x)+1$$ (۶)

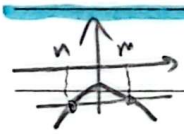
$$\rightarrow \frac{\sin^2x-2\sin x+1-\sin^2x-2\sin x-1}{(\sin x+1)^2}=\frac{-4\sin x}{(\sin x+1)^2}=x\,g(x)$$

$$g(x)=\frac{-4\sin x}{x(\sin x+1)^2}\longrightarrow \lim_{x\to 0} g(x) \xrightarrow[\text{اولیه}]{\text{حد گیری}} \lim_{x\to 0}\frac{-4x}{x(x+1)^2}=\frac{0}{0}=$$ مبهم

$$\lim_{x\to 0}\frac{-4}{(x+1)^2}\xrightarrow{x=0}\frac{-4}{1}=(-4)$$

---

(۷) ✱ قرینه یک خط نسبت به محور xها ← $y_{x=1}$ ← $y=-x^2-1$



✱ حاصل ضرب ۲ شیب ۲ خط عمود = -1 و y ۲ نقطه تماس کیست؟

$$y'=-2x\longrightarrow -2m\alpha-2n=4m\alpha n=-1\rightarrow m\alpha n=-\frac{1}{4}$$

$$y=-x^2-1\longrightarrow -m^2-1=-n^2-1\rightarrow m^2=n^2\rightarrow m=\pm n$$

✱ باتوجه به اینکه حاصل ضرب $m\times n$ عددی منفی است پس $m=-n$

$$\rightarrow n\times -n=-n^2=-\frac{1}{4}\longrightarrow n=\frac{1}{2},\ m=-\frac{1}{2}\rightarrow y=-\frac{1}{4}-1=-\frac{5}{4}$$

فاصله تا مبدا $=\frac{|0+0-\frac{5}{4}|}{\sqrt{1}}=\frac{5}{4}=d$ ← $y=-\frac{5}{4}$ ← d خط

---

(۸) ✱ نقطه $(0,0)$ روی تابع $f(x)$ مشتق ندارد یعنی نقطه شکستگی یا زاویه ایجاد میکند.

$$x=a\longrightarrow f(a)=2\sqrt{a}\,(4a^2+4)$$

$$f'(x)=\left(\frac{1}{2\sqrt{x}}\times(4x^2+4)\right)+(2\sqrt{x}\times 8x)=\frac{4x^2+4}{\sqrt{x}}+16x\sqrt{x}=\frac{20x^2+4}{\sqrt{x}}$$

شیب $m=\frac{\Delta y}{\Delta x}\rightarrow \frac{2\sqrt{a}\,(4a^2+4)}{a}=\frac{8a^2+8}{\sqrt{a}}=m$

$x=a\rightarrow f'(a)=\frac{20a^2+4}{\sqrt{a}}\longrightarrow f'(a)=m\Rightarrow \frac{20a^2+4}{\sqrt{a}}=\frac{8a^2+8}{\sqrt{a}}$

$$\rightarrow 12a^2=4\longrightarrow \begin{cases} a=\frac{1}{2} \longrightarrow m=\frac{8\times\frac{1}{4}+8}{\sqrt{\frac{1}{2}}}=\sqrt{2}\times 8=8\sqrt{2} \Rightarrow \text{ق} \\ a=-\frac{1}{2} \text{ زیر آسمانیه ⊖ بازه ← غ ق} \end{cases}$$

SUBJECT:

Year: Month: Day:

Page : ( )

⑨ مشتق تابع $f(x)$ روی بازه $(0,+\infty)$ برابر $f'(x)$ است. معادله مماس بر نمودار تابع در نقطه به طول a را بیابید.

$x=a \rightarrow f(a)=\frac{\sqrt{a}}{-2a^2+a+1}$

$f'(x)=\frac{1}{2\sqrt{x}}\times(-2x^2+x+1)+(\sqrt{x}\times(-4x+1))\times\frac{1}{(-2x^2+x+1)^2}$

$m=\frac{\Delta y}{\Delta x} \Rightarrow \frac{\frac{\sqrt{a}}{-2a^2+a+1}}{a}=\frac{\sqrt{a}}{a(-2a^2+a+1)}=\frac{1}{\sqrt{a}(-2a^2+a+1)}$

$x=a \rightarrow f'(a)=\left(\frac{-2a^2+a+1}{2\sqrt{a}}+\frac{-4a^2+2a}{2\sqrt{a}}\right)\times\frac{1}{(-2a^2+a+1)^2}$

$m=f'(a)=\frac{4a^2-a+1}{2\sqrt{a}(-2a^2+a+1)^2}=\frac{1}{\sqrt{a}(-2a^2+a+1)} \rightarrow 4a^2-a+1=-2a^2+a+1$

$\rightarrow 8a^2-2a=0 \rightarrow \begin{cases} a=0 \rightarrow \text{ق ق ق} \\ a=\frac{1}{4} \end{cases} \rightarrow f(a)=\frac{\frac{1}{2}}{\frac{-1}{16}+\frac{1}{4}+1}=\left(\frac{8}{9}\right)\rightarrow 2$

⑩ $(fog)'(x)=f'(g(x))\times g'(x)$

$g'(x)=\frac{2x}{2\sqrt{x^2-1}} \xrightarrow{x=\frac{\sqrt{5}}{2}} \frac{\sqrt{5}}{2\sqrt{\frac{5}{4}-1}}=\frac{\sqrt{5}}{2\times\frac{1}{2}}=\sqrt{5}$

$g(x)=\frac{1}{\sqrt{x^2-1}} \xrightarrow{x=\frac{\sqrt{5}}{2}} \frac{1}{\sqrt{\frac{5}{4}-1}}=2$

$x<\frac{\sqrt{5}}{2} \rightarrow x^2<\frac{5}{4} \rightarrow x^2-1<\frac{1}{4} \rightarrow \sqrt{x^2-1}<\frac{1}{2}$

$\rightarrow \frac{1}{\sqrt{x^2-1}}>2$

$f(x)=(x\times[x^2])^2=8x^6 \rightarrow f'(x)=24x^5$

$x=2 \rightarrow f'(2)=24\times 8=192$

$\Rightarrow (fog)'(\frac{\sqrt{5}}{2})=192\times\sqrt{5}=96\sqrt{5} \rightarrow \frac{96\sqrt{5}}{-48\sqrt{5}}=-2$ برابر

Benobar