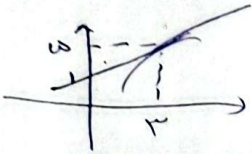


$$f'(3) = \frac{5-1}{3} = \frac{4}{3}$$



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$$f'(a) = \frac{1}{3} \quad y-1 = \frac{1}{3}(x+1) \Rightarrow y = \frac{1}{3}x + \frac{4}{3}$$

$$\sqrt{ax-1} = \frac{1}{3}x + \frac{4}{3} \Rightarrow 9ax-9 = x^2+1x+16 \Rightarrow x^2+(1-9a)x+25=0$$

$$1-9a=0 \Rightarrow a = \frac{1}{9} \quad f(x) = x \quad \checkmark$$

$$1-9a=1 \Rightarrow a=0$$

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$$f'(1) = \frac{4}{3} \quad f'(x) = \frac{(x+m)(x+3) - (x^2+3x+1)}{(x+3)^2} = \frac{1+3m-x-m}{(x+3)^2}$$

$$\frac{4+3m}{27} = \frac{4}{3} \quad m=2 \quad f(x) = \frac{x^2+3x+1}{x+3} \quad f(1) = 1 \quad u=1$$

$$y-1 = \frac{4}{3}(x-1) \Rightarrow \frac{4}{3}x + \frac{1}{3} = y \Rightarrow 4x+1-3y$$

$$y \circ f = \frac{a}{3+\sin x} = \frac{a \sin x}{(3-\sin x)(3+\sin x)} = \frac{-\sin^2 x}{3+\sin x} = (-\sin x)'$$

$$\rightarrow -\cos x \Rightarrow (y \circ f)' = \frac{1}{3} \quad \checkmark$$

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$$\left( f \left( g \left( \frac{1}{\sqrt{x}} \right) \right) \right)' \Rightarrow \frac{-1}{\sqrt{x^2+1} \sqrt{x^2+1}} = \frac{-1}{x^2+1}$$

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$$\lim_{x \rightarrow 0} g(x) = \frac{\left( \frac{-1 + \sin x}{1 + \sin x} \right)^x - 1}{x} \stackrel{\text{Hop}}{=} \frac{\left( \frac{\cos x(1 + \sin x) - \cos(-1 + \sin x)}{(1 + \sin x)^2} \right) \left( \frac{-1 + \sin x}{1 + \sin x} \right)^{x-1}}{1}$$

$$\boxed{-\frac{1}{2}}$$



$$y = a^x + 1 \xrightarrow{\text{قرینه نسبت به } x} y_1 = -(a^x + 1) = -a^x - 1 \xrightarrow{\text{شتقاق}} y'_1 = -\ln a \cdot a^x$$

خط مماس به منحنی  $y_1$  در نقطه  $A$  و خط مماس به منحنی  $y$  در نقطه  $B$  موازی می‌باشند.  $A(\alpha, B)$ ,  $B(-\alpha, \beta) \rightsquigarrow A(\frac{1}{\sqrt{e}}, \beta)$ ,  $B(-\frac{1}{\sqrt{e}}, \beta)$

$$m_{L_1} = y'_1(-\alpha) = -\ln a \cdot a^{-\alpha} = \ln a$$

$$m_{L_2} = y'_1(\alpha) = -\ln a \cdot a^{\alpha}$$

$$L_1, L_2 \text{ عمودند} \rightarrow m_{L_1} \times m_{L_2} = -1 \rightarrow \ln a \cdot \ln a \cdot a^{\alpha} = -1 \rightarrow \ln a = -\frac{1}{a^{\alpha}} \rightarrow \alpha = \pm \frac{1}{\sqrt{e}}$$

$$\text{نقطه } B \text{ را در } \alpha = \frac{1}{\sqrt{e}} \text{ قرار دهیم} \rightarrow \beta = y_1\left(\frac{1}{\sqrt{e}}\right) = -\left(\frac{1}{\sqrt{e}}\right)^{\frac{1}{\sqrt{e}}} - 1 = -\frac{1}{\sqrt{e}} - 1 = -\frac{1}{\sqrt{e}} - 1 \rightarrow |\beta| = \frac{1}{\sqrt{e}} + 1$$

$$f'(x) = \frac{1}{\sqrt{x}} (2x^2 + 3) + 1 \cdot 2x \sqrt{x} = \frac{2x^2 + 3 + 4x^2}{\sqrt{x}} = \frac{6x^2 + 3}{\sqrt{x}} = \frac{2\sqrt{x}(3x^2 + 1)}{x}$$

$$\text{نقطه } x = \frac{f(x) - 0}{f'(x)} = \frac{2x^2 + 3}{\frac{2\sqrt{x}(3x^2 + 1)}{x}} = \frac{(2x^2 + 3)x}{2\sqrt{x}(3x^2 + 1)}$$

$$2x^2 + 3 = 1 \cdot x^2 + 6 \Rightarrow 12x^2 - 3 = 0 \Rightarrow x = \pm \frac{1}{2} \Rightarrow \pm \frac{1}{2} \text{ و } 0$$

$$f'(x) = \frac{1}{2\sqrt{x}} (-2x^2 + x + 1) + (2x + 1)\sqrt{x} = \frac{-2x^2 + x + 1 + 4x^2 + 2x + 1}{2\sqrt{x}} = \frac{2x^2 + 3x + 2}{2\sqrt{x}}$$

$$\text{نقطه } x = \frac{f(x) - 0}{f'(x)} = \frac{-2x^2 + x + 1}{\frac{2x^2 + 3x + 2}{2\sqrt{x}}} = \frac{(-2x^2 + x + 1) \cdot 2\sqrt{x}}{2x^2 + 3x + 2}$$

$$-2x^2 + x + 1 = 1 \cdot x^2 + 3x + 2 \Rightarrow 3x^2 + 2x + 1 = 0 \Rightarrow x = \frac{-2 \pm \sqrt{4 - 12}}{6} = \frac{-2 \pm 2i}{6} = \frac{-1 \pm i}{3}$$

$$\boxed{f\left(\frac{1}{\sqrt{e}}\right) = \frac{\sqrt{e}}{e}}$$



$$g\left(\frac{\sqrt{a}}{x}\right) \times f\left(g\left(\frac{\sqrt{a}}{x}\right)\right) = -\ln a \sqrt{a}$$

$$g(x) = \frac{x}{\sqrt{x^2 - 1}} \Rightarrow g'(x) = \frac{x \cdot (-2x) - (\sqrt{x^2 - 1}) \cdot 1}{(x^2 - 1)^2} = \frac{-2x^2 - \sqrt{x^2 - 1}}{(x^2 - 1)^2}$$

$$g(1) = \frac{1}{\sqrt{1-1}} \rightarrow g'(1) = \frac{1}{\sqrt{1-1}} = \frac{1}{0} = \infty$$

$$f'(x) = \frac{1}{\sqrt{x}} \Rightarrow f'(1) = 1$$

$$\rightarrow g'\left(\frac{\sqrt{a}}{x}\right) \times f'\left(g\left(\frac{\sqrt{a}}{x}\right)\right) = -\ln a \sqrt{a} \rightarrow \frac{-2x^2 - \sqrt{x^2 - 1}}{(x^2 - 1)^2} \times \frac{1}{\sqrt{g\left(\frac{\sqrt{a}}{x}\right)}} = -\ln a \sqrt{a}$$