نام و نام خانوادگی ....مهرانا زنبل.... پاسخنامه تشریحی تکلیف شماره ..۲۴.. کلاس ....جمعه A....

۱

$y = ax + b$ $\quad a = f'(3)$ $\quad b = 1$

$y = ax + 1 \xrightarrow{(3,5)} 5 = 3a + 1$

$4 = 3a$

$\frac{4}{3} = a$

$\boxed{f'(3) = \frac{4}{3}}$

۲

$f(x) = \sqrt{2x-1} \longrightarrow \sqrt{9} = \boxed{3 = f(5)}$

معادله خط مماس $y = \frac{1}{3}x + \frac{4}{3}$

$f(x) = \sqrt{ax-1} \longrightarrow \sqrt{ax-1} = \frac{1}{3}x + \frac{4}{3} \xrightarrow{\text{به توان ۲}} ax - 1 = \frac{1}{9}x^2 + \frac{16}{9} + \frac{8}{9}x \xrightarrow{\times 9}$

$x^2 + x(8-9a) + 25 = 0$

$\Delta = 0 \longrightarrow a = -\frac{2}{9}$ ✗

$a \longrightarrow 2$ ق ق

۳

$y = \frac{n+3x}{4} \longrightarrow y' = \frac{3}{4}$ $\quad f(x) = \frac{x^2+mx+1}{x+3}$ $\quad f'(x) = \frac{x^2+6x+3m-1}{(x+3)^2}$

$f'(1) \longrightarrow \frac{\cancel{1}+6+3m\cancel{-1}}{16} = \frac{3}{4} \longrightarrow 6+3m = 12 \longrightarrow 3m = 6 \longrightarrow m=2$

$f(1) = \frac{1+2+1}{4} = 1$ $\quad f(1) = 1$ $\quad y = \frac{n+3}{4} = 1$

$4 = n + 3 \longrightarrow n = 1$

$\boxed{m + n = 3}$

۴

$3g - f = \frac{9}{3+\sin x} - \frac{(3-\sin x)(9+\sin^2 x+3\sin x)}{(3-\sin x)(3+\sin x)} \to \frac{9-9-\sin^2 x-3\sin x}{3+\sin x} =$

$\frac{-\sin x(\sin x+3)}{\sin x+3} = -\sin x \xrightarrow{\text{مشتق}} -\cos x \xrightarrow{\frac{\pi}{3}} \boxed{-\frac{1}{2}}$

۵

$(f\circ g)' = g'(x)\, f'(g(x)) \longrightarrow f(g(x)) = ?$ $\quad |x^5| = x^5$ $\quad g(x) = \frac{1}{2x^5}$

$f\circ g = \frac{-1}{\sqrt[5]{\frac{1}{2x^5}+\frac{1}{2x^5}}} = \frac{-1}{\sqrt[5]{\frac{1}{x^5}}} = -x \xrightarrow{\text{مشتق}} \boxed{-1}$

$\left(\frac{\sin x-1}{\sin x+1}\right)^2 = x(g(x))+1 \rightarrow \frac{\left(\frac{\sin x-1}{\sin x+1}\right)^2-1}{x} = g(x)$ $\lim_{x\to 0}\frac{\left(\frac{\sin x-1}{\sin x+1}\right)^2-1}{x} = \frac{0}{0} \xrightarrow{\text{هم‌ارزی}} \lim_{x\to 0}\frac{\left(\frac{x-1}{x+1}\right)^2-1}{x} \xrightarrow{\text{Hop}} 2\left(\frac{x-1}{x+1}\right)\left(\frac{-2}{(x+1)^2}\right)$ $x=0$ $\boxed{-4} = 2\left(\frac{-1}{1}\right)\left(\frac{-2}{1}\right)$	۶ ۳۶
$d = k$ عرض y است به محورها $\rightarrow y_2 = -x^2-1 = k \rightarrow -k-1 = x^2$ $\pm\sqrt{-k-1}$ $y'_2 = 2x \rightarrow -2\left(\sqrt{-k-1}\right)\left(\pm\sqrt{-k-1}\right) = -1$ $-k-1 = \frac{1}{4} \longrightarrow -k = 1{,}25 \rightarrow \boxed{k = -1{,}25}$ $k = -1{,}25$ غلط از نظر [illegible] $\boxed{1{,}25}$	۷ ۵۹
$d: y = ax \qquad f'(x) = 2\left(\frac{1}{2\sqrt{x}}\left(4x^2+3\right)+\sqrt{x}\,(8x)\right) = \frac{4x^2+3}{\sqrt{x}} + \frac{16x^2}{\sqrt{x}}$ $\rightarrow f'(x) = \frac{20x^2+3}{\sqrt{x}} = a = \frac{\left(20\left(\frac{1}{4}\right)+3\right)(\sqrt{2})}{} \rightarrow 8\sqrt{2} = a$ $ax = \frac{8\sqrt{2}}{2} = \boxed{4\sqrt{2}}$ $2\sqrt{x}\,(4x^2+3) = ax \rightarrow a = \frac{2\sqrt{x}(4x^2+3)}{\cancel{x}} = \frac{20x^2+3}{\sqrt{x}}$ $x = \pm\frac{1}{2} \rightarrow \boxed{x = \frac{1}{2}}$ غ ق ق $\frac{\cancel{8x^2+6}}{\cancel{2(4x^2+3)}} = \frac{12x^2}{\cancel{20x^2+3}} \rightarrow \frac{1}{4} = x^2$	۸
$d: y = ax \qquad f'(x) = \frac{\frac{1}{2\sqrt{x}}\left(-2x^2+x+1\right) - (-4x+1)(\sqrt{x})}{\left(-2x^2+x+1\right)^2} = a$ $\frac{\cancel{\sqrt{x}}}{x\cancel{(-2x^2+x+1)}} = a = \frac{6x^2-x+1}{\cancel{(-2x^2+x+1)^2}(2\sqrt{x})} \rightarrow -4x^2+2x+1 = 6x^2-x+1$ $10x^2-3x-1=0$ $x^2-3x-10=0$ $(x-5)(x+2)=$ $a = \sqrt{2} \qquad ax = \sqrt{2}\times\frac{1}{2} = \boxed{\frac{\sqrt{2}}{2}}$ ق ق $+0{,}5$ $+0{,}5$ $-\frac{2}{10} = -2$ غ ق ق	۹
$(f\circ g)' \rightarrow g'(x)\left(f'(g(x))\right)$ $\qquad g(x) \xrightarrow{\sqrt{5}/2} \frac{1}{\frac{1}{2}} = 2$ $g(x) = (x^2-1)^{-\frac{1}{2}} \longrightarrow g'(x) = -\frac{1}{2}(2x)(x^2-1)^{-\frac{3}{2}}$ $\frac{\cancel{2}\cancel{\sqrt{5}}\times\cancel{24}\left(\frac{5}{4}\right)}{\cancel{-48\sqrt{5}}} = -\frac{5}{2} = \boxed{-2{,}5}$ $g'\left(\frac{\sqrt{5}}{2}\right) = \frac{1}{2}(-\sqrt{5})\left(\frac{1}{4}\right)^{-\frac{3}{2}} =$ $\frac{\sqrt{5}}{2}\times 8 = 4\sqrt{5}$ $f \rightarrow (2x)^3 \xrightarrow{f'} 3(2)(4x^2) = 24x^2$	۱۰