۱- $m = \dfrac{5-1}{3-0} = \dfrac{4}{3} = f'(3)$ ✓ ۲

۲- $m = \dfrac{1-2}{-1-2} = \dfrac{-1}{-3} = \dfrac{1}{3} \Rightarrow 1 = \dfrac{1}{3} \times (-1) + b \Rightarrow 1 = -\dfrac{1}{3} + b \Rightarrow b = \dfrac{4}{3}$

$y = \dfrac{1}{3}x + \dfrac{4}{3}$

$\sqrt{ax-1} = \dfrac{1}{3}x + \dfrac{4}{3} \Rightarrow ax - 1 = \dfrac{1}{9}x^2 + \dfrac{16}{9} + \dfrac{8}{9}x \Rightarrow 9ax - 9 = x^2 + 16 + 8x$

$\Rightarrow x^2 + 25 + (8-9a)x = 0 \Rightarrow \Delta = 0 \Rightarrow (8-9a)^2 - 4(25) = 0 \Rightarrow 8 - 9a = 10$

$9a = -2 \quad a = -\dfrac{2}{9}$

$8 - 9a = -10 \Rightarrow a = 2$

$a = -\dfrac{2}{9} \Rightarrow f(x) = \sqrt{-\dfrac{2}{9}x - 1} \Rightarrow f(5) = \sqrt{-\dfrac{10}{9} - 1}$ غ‌ق‌ق

۲

$a = 2 \Rightarrow f(x) = \sqrt{2x-1} \Rightarrow f(5) = 3$ ✓

۳- $4y - 3x = n \Rightarrow y = \dfrac{3}{4}x + \dfrac{n}{4} \Rightarrow$ شیب $= \dfrac{3}{4}$

$y = \dfrac{x^2+mx+1}{x+3} \Rightarrow y' = \dfrac{(2x+m)(x+3) - (1)\times(x^2+mx+1)}{(x+3)^2}$

$\xrightarrow{x=1} \dfrac{(2+m)\times 4 - (2+m)}{16} \Rightarrow \dfrac{3(2+m)}{16} = \dfrac{3}{4} \Rightarrow 6 + 3m = 12 \Rightarrow m = 2$

۲

$y = \dfrac{x^2+2x+1}{x+3} \xrightarrow{x=1} \dfrac{4}{4} = 1 \qquad y = \dfrac{3}{4} \times x + \dfrac{n}{4} \Rightarrow 1 = \dfrac{3}{4} \times 1 + \dfrac{n}{4} \Rightarrow n = 1$

$m + n = 2 + 1 = 3$ ✓

۴- $f(x) = \dfrac{27 - \sin^3 x}{9 - \sin^2 x} = \dfrac{\cancel{(3 - \sin x)}(9 + \sin^2 x + 3\sin x)}{\cancel{(3 - \sin x)}(3 + \sin x)} = \dfrac{\sin^2 x + 3\sin x + 9}{\sin x + 3}$

$3g(x) = \dfrac{9}{3+\sin x} \qquad (3g - f)(x) = \dfrac{9 - (\sin^2 x + 3\sin x + 9)}{3 + \sin x} = \dfrac{-\sin^2 x - 3\sin x}{3 + \sin x}$ ۲

$= \dfrac{-\sin x(\cancel{\sin x + 3})}{\cancel{3 + \sin x}} = -\sin x$

$\Rightarrow (3g - f)'(x) = -\cos x \Rightarrow -\cos\dfrac{5\pi}{3} = -\dfrac{1}{2}$ ✓

۵-

$g'(\sqrt[5]{3})\cdot f'(g(\sqrt[5]{3})) = (f\circ g)'(\sqrt[5]{3})$ $\quad f\circ g(x) = \dfrac{-1}{\sqrt[5]{\dfrac{1}{x^5+|x^5|}+\left|\dfrac{1}{x^5+|x^5|}\right|}}$

۴

$= \dfrac{-1}{\sqrt[5]{\dfrac{1}{2x^5}+\dfrac{1}{2x^5}}} = \dfrac{-1}{\sqrt[5]{\dfrac{2}{2x^5}}} = \dfrac{-1}{\sqrt[5]{\dfrac{1}{x^5}}} = \dfrac{-1}{\dfrac{1}{x}} = -x \Rightarrow (f\circ g')(x) = -1$

$(f\circ g)'(\sqrt[5]{3}) = -1$

۶-

$g(x)=\dfrac{f(x)-1}{x}$ $\quad \lim\limits_{x\to 0} g(x) = \lim\limits_{x\to 0}\dfrac{f(x)-1}{x} = \lim\limits_{x\to 0}\dfrac{\left(\dfrac{-1+\sin x}{1+\sin x}\right)^2-1}{x}$

۴

$\xrightarrow{\text{هم‌ارزی}} \dfrac{\left(\dfrac{-1+x}{1+x}\right)^2-1}{x} = \dfrac{\dfrac{+1+x^2-2x}{x^2+1+2x}-1}{x} = \dfrac{\cancel{x^2}+\cancel{1}-2x-\cancel{x^2}-\cancel{1}-2x}{\dfrac{x^2+1+2x}{x}} = \dfrac{-4x}{x(x^2+1+2x)}$

$= \dfrac{-4}{(x+1)^2} \xrightarrow{x=0} = -4$

=

۷-

$y=x^2+1 \Rightarrow y=-x^2-1 \Rightarrow -x^2-1=k \Rightarrow x^2=-1-k$

$y=k \qquad x=\pm\sqrt{-1-k}$

$y'=2x \Rightarrow y' \Rightarrow 2\sqrt{-1-k}$, $\; -2\sqrt{-1-k}$ $\qquad -2\sqrt{-1-k}\times 2\sqrt{-1-k} = -4(-1-k) = -1 \Rightarrow -1-k=\frac{1}{4}$

$-k=\frac{5}{4}$

$k=-\frac{5}{4}$

۲

۸-

$2\times\dfrac{1}{2\sqrt{x}}(4x^2+3)+8x\times\sqrt{x} \xrightarrow{x=0}$ تن

۵ (جواب پایین صفحه)

$f'(x)=\dfrac{\dfrac{1}{2\sqrt{x}}(-2x^2+x+1)-(-4x+1)(\sqrt{x})}{(-2x^2+x+1)^2} = \dfrac{\dfrac{1}{2\sqrt{x}}\left[(-2x^2+x+1)-(-4x+1)(2x)\right]}{(-2x^2+x+1)^2}$

۱

$= \dfrac{-2x^2+x+1+8x^2-2x}{2\sqrt{x}(-2x^2+x+1)^2} = \dfrac{6x^2-x+1}{2\sqrt{x}(-2x^2+x+1)^2} =$ خط $\rightarrow y=ax$ $\qquad A(\alpha, a\alpha)$

$f(\alpha)=\dfrac{\sqrt{\alpha}}{-2\alpha^2+\alpha+1}=a\alpha \rightarrow a\sqrt{\alpha}(-2\alpha^2+\alpha+1)=1 \rightarrow -2a\alpha^{\frac{5}{2}}+a\alpha^{\frac{1}{2}}+a\alpha^{\frac{3}{2}}=1$ $\qquad f(\alpha)=\dfrac{\sqrt{\frac{1}{2}}}{-2(\frac{1}{2})^2+\frac{1}{2}+1}=\dfrac{\sqrt{2}}{2}$

$\xrightarrow{\text{مشتق}} -5a\alpha^{\frac{3}{2}}+\frac{3}{2}a\alpha^{\frac{1}{2}}+\frac{1}{2}a\alpha^{-\frac{1}{2}}=0 \xrightarrow[\times 2\sqrt{\alpha}]{\div a} -10\alpha^2+3\alpha+1=0 \rightarrow \begin{cases}\alpha=\frac{-1}{5}\\ \alpha=\frac{1}{2}\end{cases}$

Arman

$fog(x) = \left(\frac{1}{\sqrt{x^2-1}}\left[\frac{1}{\sqrt{x^2-1}}\right]\right)^{۳} = \left(\frac{۲}{\sqrt{x^2-1}}\right)^{۳} = \frac{۸}{(x^{۲}-1)^{\frac{۳}{۲}}} = ۸ \times (x^{۲}-1)^{-\frac{۳}{۲}}$ ۱۰ـ

$x = \frac{\sqrt{۵}}{۲} \Rightarrow \left[\frac{1}{\sqrt{x^{۲}-1}}\right] = \left[\frac{1}{\sqrt{\frac{۵}{۴}-1}}\right] = \left[\frac{1}{\frac{1}{۲}}\right] = ۲$

$۸ \times -\frac{۳}{۲} \times (x^{۲}-1)^{-\frac{۵}{۲}} \times ۲x \xrightarrow{x=\frac{\sqrt{۵}}{۲}} \cancel{۸}^{۴} \times -\frac{۳}{\cancel{۲}_{1}} \times \left(\frac{1}{۴}\right)^{-\frac{۵}{۲}} \times \sqrt{۵} = -۱۲ \times ۳۲\sqrt{۵}$

$\frac{-\cancel{۱۲}^{1} \times ۳۲\sqrt{۵}}{-\frac{\cancel{۴۸}\sqrt{۵}}{۴}} = ۸$ ✓

۲

سوال ۱۸

$f(x) = ۲\sqrt{x}(۴x^{۲}+۳) = ۸x^{۲}\sqrt{x} + ۶\sqrt{x} \longrightarrow f'(x) = ۲۰x\sqrt{x} + \frac{۳}{\sqrt{x}} = \frac{۲۰x^{۲}+۳}{\sqrt{x}}$

$y - ۲\sqrt{\alpha}(۴\alpha^{۲}+۳) = \frac{۲۰\alpha^{۲}+۳}{\sqrt{\alpha}}(x-\alpha) \xrightarrow{(۰,۰)} -۲\sqrt{\alpha}(۴\alpha^{۲}+۳) = \frac{۲۰\alpha^{۲}+۳}{\sqrt{\alpha}}(-\alpha)$

$\longrightarrow ۲(۴\alpha^{۲}+۳) = ۲۰\alpha^{۲}+۳ \longrightarrow ۱۲\alpha^{۲} = ۳ \longrightarrow \alpha^{۲} = \frac{1}{۴}$

$m = \frac{۲۰(\frac{1}{۴})+۳}{\sqrt{\frac{1}{۴}}} = ۸\sqrt{۴}$