① $f(x) = x - \frac{a}{x}$ $[1, 3]$

$1 \to 1 - a$

$3 \to 1 - \frac{a}{3}$

$\frac{1 - \frac{a}{3} - 1 + a}{2} = \frac{a}{3}$

$f'(x) = \frac{a}{x^2} \to \frac{a}{x^2} = \frac{a}{3} \to x^2 = 3 \to x = \pm\sqrt{3}$ $\begin{cases} x = -\sqrt{3} & \times \\ x = \sqrt{3} & \checkmark \end{cases}$

② $y = 2ax^2 - 5x + 18a$ → نقطة على المحور x $\to x = y$

$y' = 4ax - 5$ $\quad 2ax^2 - 5x + 18a = x$

$\Delta = 0 \to 49 - 4(18)(2a) = 0 \to a = \pm\frac{1}{2} \to a = -\frac{1}{2}$ ✓

③ $y = x^3 - 12x + 2$

$y' = 3x^2 - 12 \to x^2 = 4 \to x = \pm 2$

	-2		2	
y'	$+$	$-$	$+$	
y	↗	↘	↗	
		min		

$x = 2 \to (8) - 12(2) + 2 = -14$ ✓

④ $y = x^3 + ax^2 - 2bx - 4$ $\quad 0, -2$ $\quad y = x^3 + 2x^2 - 4$

$y' = 3x^2 + 2ax - 2b$ $\quad (-2, 0)\ (0, -4)$

$0 \to -2b = 0 \to b = 0$

$-2 \to 3(4) = 4a \to a = 3$

نصف قطر: $\sqrt{4, 14} = \sqrt{20}$ ✓

⑤ $f(x) = x^2 - 3|x|$

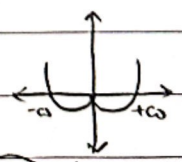
$$f(x) = \begin{cases} x^2 - 3x = x(x-3) & x \geq 0 \\ x^2 + 3x = x(x+3) & x < 0 \end{cases}$$

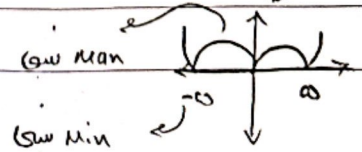
$0 \to 0$ $\quad -3 \to 0$

$|f(x)|$ ⇝ max, min

max: m, 2 $\quad$ min: n, 3

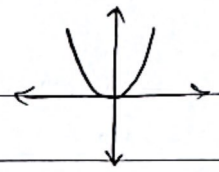
$\frac{n}{m} = \frac{3}{2}$



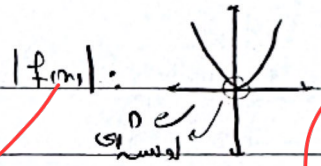


s.a.m

④ $f(x)=\begin{cases} x^2+3x = x(x+3) & x\geq 0 \\ x(x-3) & x<0 \end{cases}$ (0, −3)



$|f(x)|$:



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⑤ $[0,a]$ ضابطه $\rightarrow \sqrt[3]{x^2}(-x+a)$ $\rightarrow f'(x)=\frac{2(a-x)}{\sqrt[3]{x}}-\sqrt[3]{x^2}$ $\rightarrow y_x=-2x+2a \rightarrow x=\frac{2a}{5}$

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$f(x_{max})=1.5 \rightarrow f(\frac{2}{5}a)=\frac{2}{3} \rightarrow \sqrt[3]{\frac{4}{125}a^2}\left(\frac{2}{5}a-a\right)=\frac{2}{3} \rightarrow a\times\sqrt[3]{\frac{4}{125}a^2}=\frac{5}{2}$

$a^{\frac{2}{3}}\times\frac{3}{5}a=\frac{3}{2} \rightarrow a^{\frac{5}{3}}=\frac{5}{2} \rightarrow a=\sqrt[5]{\frac{125}{32}}$

توان ۳ $\rightarrow a^2\times\frac{4}{125}a^2=\frac{125}{8} \rightarrow a^5=\frac{125}{8}\times\frac{32}{4}=(\frac{5}{2})^5 \rightarrow a=\frac{5}{2}=2.5$

⑥ $f(x)=\begin{cases}\sqrt{x^2-x} & x\geq 1 \\ \sqrt{-x^2-x} & x<1\end{cases}$

$\frac{km+n}{k-n}=\frac{f(1)+0}{2}=1$

$f'(x)=\begin{cases}\frac{2x-1}{2\sqrt{x^2-x}} & x>1 \\ \frac{-2x-1}{2\sqrt{-x^2-x}} & x<0\end{cases}$

$f'(x) \rightarrow 1_+ - 1_-$

$f'(x)=0 \rightarrow -\frac{1}{2}$

$k=4$

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$f'(-\frac{1}{2})>f'_+(-\frac{1}{2})<0$ $\xrightarrow[\text{می‌نیمم}]{\text{مشتق‌پذیر}}$ $-\frac{1}{2} \rightarrow m=1$

سوال ۹

$f(x)<0 \rightarrow ad-bc<0 \rightarrow m^2-m-2<0 \rightarrow (m-2)(m+1)<0 \rightarrow -1<m<2 \ , \ m\neq 2 \rightarrow -1<m<2 \ (I)$

$\text{ریشه مخرج}=x=1-m<1 \rightarrow m>0 \ (II)$ $\quad (I)\cap(II) \rightarrow m=0,1$

سوال ۱۰

$D_{f(x)}=1-x|x|=0 \rightarrow x|x|=1 \rightarrow \begin{cases} x\geq 0 & x^2=1 \rightarrow x=1 \ \checkmark \\ x<0 & -x^2=1 \rightarrow x^2=-1 \ X \end{cases} \rightarrow D_f=\mathbb{R}-\{1\}$

مشتق تابع $\begin{cases} x>0 \rightarrow f'(x)=\frac{1-x^2+2x^2}{(1-x^2)^2}=\frac{x^2+1}{(1-x^2)^2} \rightarrow x^2=-1 \ X \\ x<0 \rightarrow f'(x)=\frac{1+x^2-2x^2}{(1+x^2)^2}=\frac{1-x^2}{(1+x^2)^2} \rightarrow x^2=1 \rightarrow x=-1 \ \checkmark \end{cases}$

$\rightarrow$ یک نقطه بحرانی