۱)

$$\frac{1-\frac{a}{3}-(1-a)}{2}=\frac{a}{x^2}\rightarrow\frac{\frac{2a}{3}}{2}=\frac{a}{x^2}\rightarrow\frac{a}{3}=\frac{a}{x^2}\rightarrow x=\pm\sqrt{3}\rightarrow x=+\sqrt{3}$$

غ ق ق

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۲)

$$y=2ax^2-5x+18a\rightarrow y'=4ax-5\xrightarrow[y=x]{\text{مماس}}4a=1\rightarrow a=\frac{1}{4}$$

$$ax^2-3x+9a=0\xrightarrow{\Delta=0}9-4(a)(9a)=0\rightarrow 9-36a^2=0\rightarrow a^2=\frac{1}{4}\rightarrow a=\pm\frac{1}{2}\rightarrow a=\frac{-1}{2}$$

$a=\frac{1}{2}\rightarrow$ نقطه تماس $\rightarrow x^2-6x+9=(x-3)^2=0\rightarrow$ ریشه مثبت غ ق ق

۱.۵

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۳)

$$y=x^3-12x+2\rightarrow y'=3x^2-12\rightarrow 3x^2=12\rightarrow x=\pm 2$$

		-2		2	
y'	+	0	−	0	+
y	↗	max	↘	min	↗

min نسبی $=(2,-14)$

مقدار min نسبی

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۴)

$$y=x^3+ax^2-2bx-4\rightarrow y'=3x^2+2ax-2b\xrightarrow{x=0}-2b=0\rightarrow b=0$$

$$\xrightarrow{x=-2}12-4a=0\rightarrow a=3$$

اکسترمم‌ها: $(0,-4)$, $(-2,0)$ $\rightarrow$ فاصله $=\sqrt{4+16}=2\sqrt{5}$

۲

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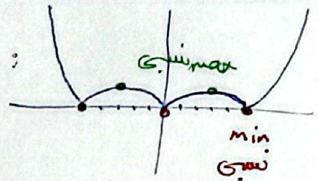
۵)

$|f(x)|$:



$$m=2,\quad n=3\rightarrow\frac{n}{m}=\frac{3}{2}$$

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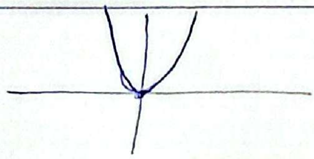
۶)

$f(x)=|f(x)|$



$\rightarrow$ ۱ نقطه بحرانی دارد

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۷)

$$f(x)=\left|x^{\frac{5}{3}}-ax^{\frac{2}{3}}\right|\rightarrow f'(x)=\frac{5}{3}x^{\frac{2}{3}}-\frac{2a}{3}x^{-\frac{1}{3}}\rightarrow$$

$$\frac{1}{3}x^{-\frac{1}{3}}(5x-2a)=0\rightarrow x=0,\quad x=\frac{2a}{5}$$

$$f\left(\frac{2a}{5}\right)=\frac{3}{2}\rightarrow\sqrt[3]{\frac{4a^2}{25}}\left|\frac{2a}{5}-a\right|=\frac{3}{2}\rightarrow\frac{4a^2}{25}\times\frac{27a^3}{125}=\frac{27}{8}\rightarrow a^5=\left(\frac{5}{2}\right)^5\rightarrow\boxed{a=\frac{5}{2}}$$

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۸)

$$f(x)=\begin{cases}\sqrt{x^2-x} & x\geq 0\\ \sqrt{-x^2-x} & x<0\end{cases}\rightarrow f'(x)=\begin{cases}\frac{2x-1}{2\sqrt{x^2-x}} & x>1\\ \frac{-2x-1}{2\sqrt{-x^2-x}} & -1<x<0\end{cases}$$

$$\rightarrow\frac{2x-1}{2\sqrt{x^2-x}}=0\rightarrow x=\frac{1}{2}\ ✗$$

$$\rightarrow -2x=1\rightarrow x=-\frac{1}{2}\ ✓$$

	-1		$-\frac{1}{2}$		0
y'		+	0	−	
y		↗	max	↘	

$$\frac{km+n}{k-n}=\frac{4}{4}=\boxed{1}$$

لازمه max نسبی $\rightarrow m=1$

۴ نقطه بحرانی: $\{-\frac{1}{2},-1,0,1\}\rightarrow k=4$

نقطه min نسبی نداره $\rightarrow n=0$

۲

$$y' = \frac{m^2 - m - 2}{(x+m-1)^2} \leqslant 0 \longrightarrow (m-2)(m+1) \leqslant 0 \qquad \begin{array}{ccccc} & -1 & & 2 & \\ + & 0 & - & 0 & + \end{array} \qquad \text{(9} \;\; \textcircled{1,5}$$

$$\to m \in [-1, 2)$$

۳ مقدار صحیح

ریشه مخرج $= x = 1 - m \leqslant 1 \rightarrow m \geqslant 0 \quad (\mathrm{II})$ $\qquad (\mathrm{I}) \wedge (\mathrm{II}) \rightarrow m = 0, 1$

$$f(x) = \frac{x}{1 - x|x|} \longrightarrow f(x) \begin{cases} \dfrac{x}{1-x^2} & x \geqslant 0 \\ \dfrac{x}{1+x^2} & x < 0 \end{cases} \longrightarrow f'(x) = \begin{cases} \dfrac{x^2+1}{(1-x^2)^2} & x \geqslant 0 \\ \dfrac{1-x^2}{(1+x^2)^2} & x < 0 \end{cases} \qquad 10$$

1

$$f'(x) = 0 \longrightarrow 1 - x^2 = 0 \longrightarrow x = \pm 1 \longrightarrow \text{۲ نقطه بحرانی}$$

$$D_{f(x)} = 1 - x|x| = 0 \longrightarrow x|x| = 1 \longrightarrow \begin{cases} x \geqslant 0 & x^2 = 1 \rightarrow x = 1 \;\checkmark \\ x \leqslant 0 & -x^2 = 1 \rightarrow x^2 = -1 \;\times \end{cases} \longrightarrow D_f = \mathbb{R} - \{1\}$$

مشتق تابع
$$\begin{cases} x \geqslant 0 \rightarrow f'(x) = \dfrac{1 - x^2 + 2x^2}{(1-x^2)^2} = \dfrac{x^2+1}{(1-x^2)^2} \rightarrow x^2 = -1 \;\times \\ x < 0 \rightarrow f'(x) = \dfrac{1 + x^2 - 2x^2}{(1+x^2)^2} = \dfrac{1-x^2}{(1+x^2)^2} \rightarrow x^2 = 1 \rightarrow x = -1 \;\checkmark \end{cases}$$

$\rightarrow$ یک نقطه بحرانی