Subject
Year Month Day

۱۰

فاطمه سلطان اکبری

$y' = 3x^2 + 2ax - 2b \Rightarrow$ ریشه $\Rightarrow x = 0$ (۴)

$x = -2$

مرکز ریشه $\rightarrow S = -2 + 0 = \frac{-2a}{3} \Rightarrow -6 = -2a \rightarrow a = 3$

ضرب ریشه $\rightarrow p = -2 \times 0 = \frac{-2b}{3} \Rightarrow b = 0$ (۲)

$y = x^3 + 3x^2 - 4 \Rightarrow \begin{cases} x = 0 \Rightarrow y = -4 \Rightarrow A(0,-4) \\ x = -2 \Rightarrow y = -8 + 12 - 4 = 0 \end{cases}$

$B(-2,0)$

$AB = \sqrt{(-2-0)^2 + (0+4)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$ ✓

$y' \leq 0$

$$y = \frac{mx + 2}{x - 1 + m} \Rightarrow y' = \frac{m(x-1) - 2}{(x - 1 + m)^2} = \frac{m^2 - m - 2}{(x - 1 + m)^2} \leq 0$$

$$\Rightarrow m^2 - m - 2 \leq 0 \Rightarrow -1 \leq m \leq 2$$

$$1 - m \leq 1 \Rightarrow 0 \leq m$$

$$\Rightarrow m = 1$$

$$m \geq 0$$

(۲)

Subject

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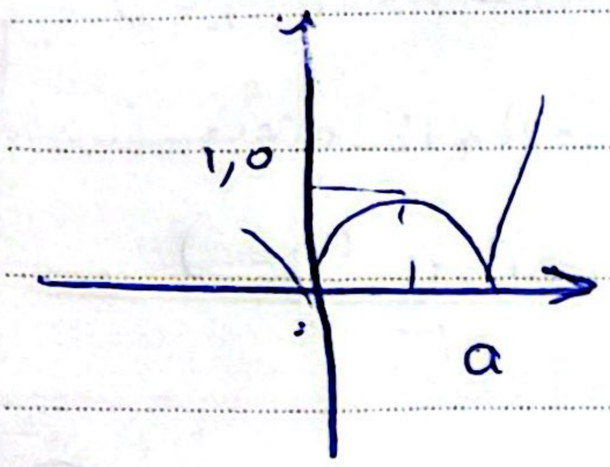
$f'(x)=0 \Rightarrow f'(x)=3x^2-12=0$ (۳)

$\Rightarrow x^2=4 \Rightarrow x=\pm 2$ (۲)

x	$-\infty$	-2		2		$+\infty$
f'		$+$	0	$-$	0	$+$
f		↗		↘		↗

$f(2)=2^3-12+2+2=8-24$

$\Rightarrow =-14$ ✓



$f(x)=-\sqrt[3]{x^2}(x-a)$ (۴)

$f'(x)=-\frac{2(x-a)}{3\sqrt[3]{x}}+\sqrt[3]{x^2}=0$ (۲)

$\Rightarrow -2x+2a=3x \Rightarrow x=\frac{2}{5}a$

$f(x)=-\sqrt[3]{\frac{4}{25}a^2}\left(-\frac{3a}{5}\right)=1,0$

$\Rightarrow a=2,0$ ✓

$\frac{f(3)-f(1)}{2}=\frac{1-\frac{a}{3}-1+\frac{a}{1}}{2}=\frac{\frac{2a}{3}}{2}=\frac{a}{3}$ (۱)

$f'(x)=\frac{a}{3x^2} \Rightarrow \frac{a}{3}=\frac{a}{3x^2} \Rightarrow x^2=3 \Rightarrow x=\sqrt{3}$ (۲) ✓

سوال ۲)

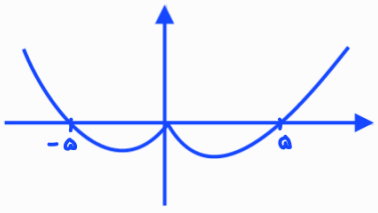
$6ax^2 - 5x + 1 = x \longrightarrow 6ax^2 - 6x + 1 = 0 \xrightarrow{\div 2} 3ax^2 - 3x + 1 = 0$

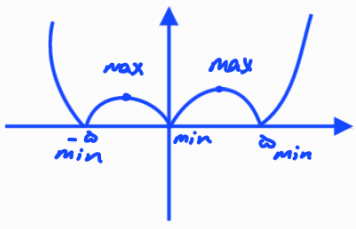
$3ax^2 - 3x + 1 = 0 \xrightarrow{\Delta = 0} 9 - 4(3a)(1) = 0 \longrightarrow 9 - 12a = 0 \longrightarrow a = \frac{3}{12} = \frac{1}{4} \longrightarrow a = \pm\frac{1}{2} \longrightarrow a = \frac{-1}{2}$

$a = \frac{1}{4} \rightarrow$ تقاطع مماسی $\rightarrow x^2 - 6x + 9 = (x-3)^2 = 0 \rightarrow$ ریشه مضاعف

سوال ۵)

$$f(x) = x^2 - 5|x| = \begin{cases} x^2 - 5x & x \geq 0 \quad (I) \\ x^2 + 5x & x < 0 \quad (II) \end{cases}$$

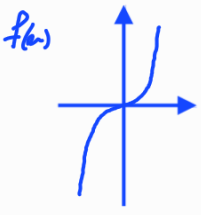


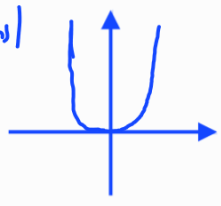


$n = 3$, $m = 2$ $\rightarrow \frac{n}{m} = \frac{3}{2}$

سوال ۶)

$$f(x) = \begin{cases} x^2 + 3x & x \geq 0 \\ -x^2 + 3x & x < 0 \end{cases} \longrightarrow f'(x) = \begin{cases} 3x + 3 & x > 0 \\ -3x + 3 & x < 0 \end{cases} \longrightarrow f'_+(0) = f'_-(0) = 3$$

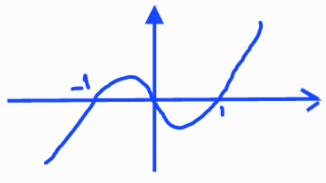


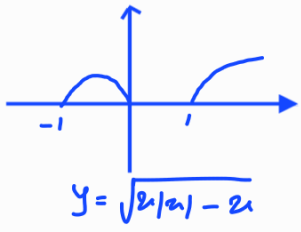


یک نقطه بحرانی $x = 0$

سوال ۸)

$$y = |x|x - x \rightarrow \begin{cases} x^2 - x & x \geq 0 \quad (I) \\ -x^2 - x & x < 0 \quad (II) \end{cases}$$





(تعداد نقاط بحرانی) $K = 4$ و (تعداد max) $m = 1$ و (تعداد min) $n = 0$

$$\frac{Km + n}{K - n} = \frac{4 \times 1 - 0}{4 - 0} = \frac{4}{4} = 1$$

سوال ۱۰)

$D_{f(x)} = 1 - x|x| = 0 \rightarrow x|x| = 1 \rightarrow \begin{cases} x \geq 0 & x^2 = 1 \rightarrow x = 1 \checkmark \\ x < 0 & -x^2 = 1 \rightarrow x^2 = -1 \times \end{cases} \rightarrow D_f = \mathbb{R} - \{1\}$

مشتق تابع $\begin{cases} x > 0 \rightarrow f'(x) = \frac{1 - x^2 + 2x^2}{(1-x^2)^2} = \frac{x^2 + 1}{(1-x^2)^2} \rightarrow x^2 = -1 \times \\ x < 0 \rightarrow f'(x) = \frac{1 + x^2 - 2x^2}{(1+x^2)^2} = \frac{1 - x^2}{(1+x^2)^2} \rightarrow x^2 = 1 \rightarrow x = -1 \checkmark \end{cases}$

یک نقطه بحرانی