«مبینا هاشمی الحسینی» ۱۸/۵ «دوازدهم تجربی A»

۱- $f(x) = 1 - \frac{a}{x} \quad [1,3]$ (۱/۵)

آهنگ متوسط $= \frac{f(3)-f(1)}{2} = \frac{1-\frac{a}{3}-1+a}{2} = \frac{\frac{2}{3}a}{2} = \frac{a}{3}$

$f'(x) = \frac{a}{x^2} \Rightarrow \frac{a}{x^2} = \frac{a}{3} \; (a \neq 0) \Rightarrow x^2 = 3 \Rightarrow x = \pm\sqrt{3}$ $\begin{cases} x = -\sqrt{3} \; ✗ \\ x = \sqrt{3} \; ✓ \end{cases}$

۲- $y = 2ax^2 - 5x + 11a \qquad y = x$

$f(x) = 4ax - 5 \qquad g'(x) = 1$ نقطه‌ای از منحنی:

$x = \left(-\frac{b}{2a}\right) < 0 \to x = \frac{5}{2a} < 0 \to a < 0$

$2ax^2 - 5x + 11a = x \leftarrow f(x) = g(x)$

$2ax^2 - 6x + 11a = 0 \to ax^2 - 3x + 9a = 0$ (۲)

$\Delta = 0 \Rightarrow 9 - 4(a)(9a) = 0$

$9 - 36a^2 = 0 \Rightarrow a^2 = \frac{1}{4} \to a = \pm\frac{1}{2} \xrightarrow{*} \boxed{a = -\frac{1}{2}}$ ✓

۳- $y = x^3 - 12x + 2 \longrightarrow$ min نسبی $= ?$

$y' = 3x^2 - 12 = 0 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$

x		-2		$+2$	
y'	$+$	0	$-$	0	$+$
y	↗	18	↘	-14	↗
		max نسبی		min نسبی	

$-8+24+2 \quad 8-24+2$

$\Rightarrow$ min نسبی $(2, -14)$ ✓ (۲)

۴- $y = x^3 + ax^2 - 2bx - 4$ ext نسبی $\begin{cases} x = 0 \\ x = -2 \end{cases}$

$y' = 3x^2 + 2ax - 2b = 0$

$x = 0 \to -2b = 0 \Rightarrow b = 0$

$x = -2 \to 12 - 4a = 0 \Rightarrow a = 3$

$\Rightarrow y' = 3x^2 + 6x = 0$

$3x(x+2) = 0$ (۲)

$\Rightarrow y = x^3 + 3x^2 - 4$

$A(0, -4)$
$B(-2, 0)$
$\Rightarrow |AB| = \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5}$ ✓

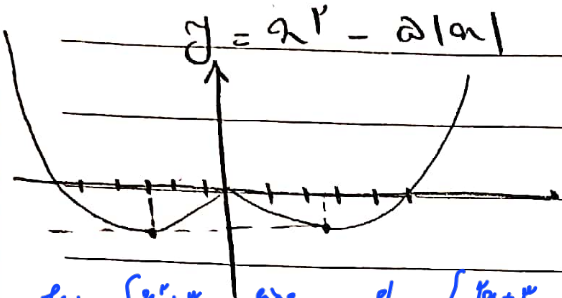
ـ۵

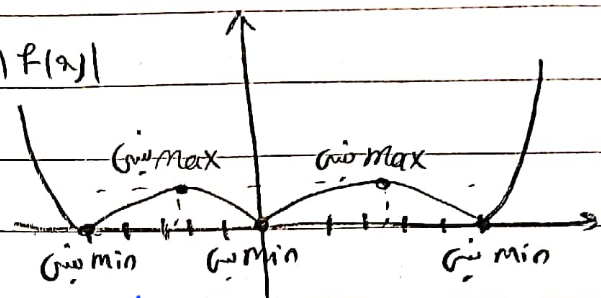
$f(x)=x^2-5|x|$ نسبی max : m

$|f(x)|=\left|x^2-5|x|\right|$ نسبی min : n $\Rightarrow \frac{n^3}{m^2}=?$ $\frac{3}{2}$

(۲)

$y=x^2-5|x|$ $|f(x)|$



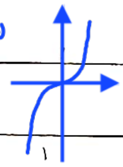


$f(x)=\begin{cases} x^2+3x & x\geq 0 \\ -x^2+3x & x<0 \end{cases} \rightarrow f'(x)=\begin{cases} 2x+3 & x\geq 0 \\ -2x+3 & x<0 \end{cases} \rightarrow f'(0^+)=f'(0^-)=3$

ـ۶   تعداد اکسترمم برای ؟

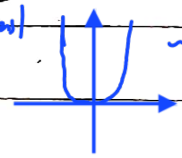
$y=|f(x)|$, $f(x)=x(|x|+3)$

$y=\left|x(|x|+3)\right|$ (۱)



$|x|=0 \Rightarrow x=0$ $\{0,-3,-\frac{3}{2}\}$ ۳ نقطه بحرانی

$x^2+3x=0 \Rightarrow x=-3$



$\rightarrow$ یک نقطه بحرانی $x=0$

$y=x^2+3x \Rightarrow y'=2x+3 \Rightarrow x=-\frac{3}{2}$

ـ۷

$f(x)=\sqrt[3]{x^2}\,|x-a|$ $[0,a]$ max مطلق $=1.5$ $a=?$

$f(x)=\sqrt[3]{x^2}\,(x-a)$

$f'(x)=\frac{2}{3\sqrt[3]{x}}(x-a)+\sqrt[3]{x^2}=0$

$\frac{2}{3\sqrt[3]{x}}(x-a)=-\sqrt[3]{x^2} \Rightarrow 2(x-a)=-3x$ (۲)

$2x-2a=-3x \Rightarrow 5x=2a \rightarrow x=\frac{2}{5}a$

$\sqrt[3]{\frac{4}{25}a^2}\,\left|\frac{-3}{5}a\right|=1.5 \rightarrow \frac{4}{25}a^2\times\frac{27}{125}a^3=\frac{27}{8}$

$a^5=\frac{25\times125}{8\times4}=\frac{5^5}{2^5} \Rightarrow a=2.5$

Scanned with CamScanner

$D_f: x^2 - x \geq 0$

$x(x-1) \geq 0$ — $+ \; 0 \; - \; 1 \; +$

۸- $f(x) = \sqrt{x|x| - x}$

m: max نسبی

n: min نسبی

K: بحرانی

$\frac{Km+n}{K-n} = ?$

$f(x) = \begin{cases} \sqrt{x^2 - x} & x \geq 1 \\ \sqrt{-x^2 - x} & x \leq 0 \end{cases}$

* تعداد نقاط بحرانی: $K=4$

* min نسبی ندارد

$f'(x) = \begin{cases} \dfrac{2x-1}{2\sqrt{x^2-x}} & x \geq 1 \\ \dfrac{-2x-1}{2\sqrt{-x^2-x}} & x \leq 0 \end{cases}$

$2x-1 \rightarrow x = \frac{1}{2}$ ✗

$2\sqrt{x^2-x} \rightarrow x = 1$ ✓

$-2x-1 \rightarrow x = -\frac{1}{2}$ ✓ → max نسبی → $m = 1$

$2\sqrt{-x^2-x} \rightarrow -x(x+1) \rightarrow x = 0$ ✓, $x = -1$ ✓

$\Rightarrow \frac{Km+n}{K-n} = \frac{4(1)}{4} = 1$

(2)

۹- $y = \frac{mx+2}{x-1+m}$ $(1, +\infty) \rightarrow$ نزولی

$y' = \frac{m(x-1+m) - (mx+2)}{(x-1+m)^2} = \frac{mx - m + m^2 - mx - 2}{(x-1+m)^2} < 0$ ($(x-1+m)^2 \rightarrow +$ همواره)

$m^2 - m - 2 < 0$

$(m-2)(m+1) < 0 \Rightarrow m \in \mathbb{Z}: \{0, 1\}$

$+ \; -1 \; - \; 2 \; +$

(2)

۱۰- $f(x) = \frac{x}{1 - x|x|}$ تعداد بحرانی؟ D_f

$x|x| \neq 1$, $x \neq 1$

~~$|x| = 0 \Rightarrow x = 0$~~ $\rightarrow D_f = \mathbb{R} - \{1\}$

$f(x) = \frac{x}{1-x^2} \Rightarrow f'(x) = \frac{1 - x^2 - (-2x)(x)}{(1-x^2)^2}$ ($+2x^2$)

$f'(x) = \frac{x^2-1}{(1-x^2)^2} \Rightarrow x = \pm 1 \Rightarrow x = -1$

(1.75)

→ یک نقطه بحرانی