صفحه کل ... «دوازدهم دفتر A» تکلیف ۲۵

۱۱٫۷۵

① $\frac{f(3)-f(1)}{3-1} = \frac{(1-\frac{a}{3})-(1-a)}{2} = \frac{\frac{2}{3}a}{2} = \frac{a}{3}$

$f(x) = 1 - \frac{a}{x} \Rightarrow f' = \frac{a}{x^2} \rightarrow \frac{a}{3} = \frac{a}{x^2} \rightarrow x = \pm\sqrt{3} \xrightarrow{x\in[1,3]} \boxed{x=\sqrt{3}}$ ✓

(۲)

② $y=-x \Rightarrow 2ax^2-5x+18a=-x \rightarrow 2ax^2-4x+18a=0$

$\Delta=0 \rightarrow 16-144a^2 \rightarrow 16=144a^2 \rightarrow a^2=\frac{1}{9} \rightarrow a=\pm\frac{1}{3} \rightarrow \boxed{a=-\frac{1}{3}}$

$a^2=\frac{1}{4} \rightarrow a=\pm\frac{1}{2} \rightarrow a=\frac{-1}{2}$

(۱٫۷۵)

③ $y=x^3-12x+2 \rightarrow 3x^2-12=0 \rightarrow x=\pm 2 \rightarrow$

| | $-2$ | | $2$ | |
|---|---|---|---|---|
| + ↗ | | − ↘ | | + ↗ |
| | | | min | |

جایگذاری $\rightarrow f(2)=(2)^3-12(2)+2=-14$

(پیشتیم نیست تو ۲) (۱٫۵)

فاصله نقاط ext $=\sqrt{(0-(-4))^2+(-2-0)^2}=\sqrt{16+4}=\sqrt{20}=2\sqrt{5}$

④ $y=x^3+ax^2-2bx-4 \rightarrow 3x^2+2ax-2b \xrightarrow{x=0} \boxed{b=0}$

↓ $x^3-3x^2-4 \;/\; -8-12-4=$ $\quad \xrightarrow{x=-2} 12-4a=0 \rightarrow \boxed{a=3}$ ✓

$(0,-4)\;(-2,-24) \rightarrow \sqrt{4+784}=\sqrt{788}=2\sqrt{197}$

(۱٫۵)

⑤ $f(x)=x^2-5|x| \xrightarrow{\text{نقاط بحرانی}}$ (1) $x=0$ , (2) $x^2-5x \rightarrow \boxed{x=5}, \boxed{x=0}$

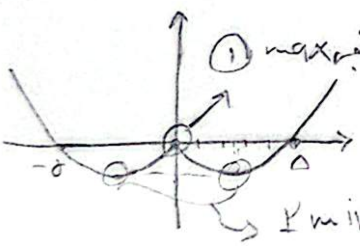
$n=3,\; m=2 \rightarrow \frac{n}{m}=\frac{3}{2}$

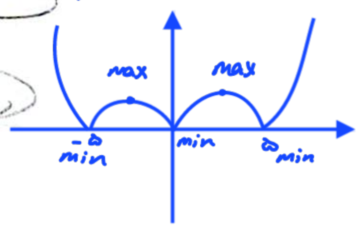
(۱)

(1) max نسبی $= m$ $\quad -\frac{b}{2a}=\frac{5}{2}=2.5 \quad +\frac{25}{4}-\frac{50}{4}=-\frac{25}{4}=-6.25$

$\rightarrow \frac{n}{m}=2$

۲ min نسبی $=n$

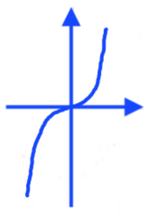





⑥

$f(x)=\begin{cases} x^2+3x & x\geq 0 \\ -x^2+3x & x<0 \end{cases} \rightarrow f'(x)=\begin{cases} 2x+3 & x\geq 0 \\ -2x+3 & x<0 \end{cases} \rightarrow f'_+(0)=f'_-(0)=3$

(۰)

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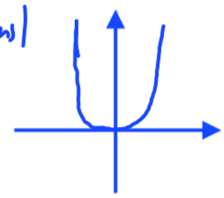


→ یک نقطه بحرانی $x=0$

(۷) $0 \le x \le a \xrightarrow{-a} -a \le x-a \le 0 \to |x-a| = a-x$

$f(x) = \sqrt[3]{x^2}\,(a-x) = a x^{\frac{2}{3}} - x^{\frac{5}{3}} \longrightarrow f'(x) = \frac{2}{3} a x^{-\frac{1}{3}} - \frac{5}{3} x^{\frac{2}{3}} = \frac{2}{3} x^{-\frac{1}{3}}\left(a - \frac{5}{2}x\right)$

$f'(x) = \dfrac{2\left(a - \frac{5}{2}x\right)}{3\sqrt[3]{x}}$ نقاط بحرانی: $a = x$ , $x = 0$ , $x = \frac{2}{5}a$ $\quad f(a) = f(0) = 0$

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$f\left(\frac{2}{5}a\right) = 1.5 \to \sqrt[3]{\left(\frac{2}{5}a\right)^2\left(a - \frac{2}{5}a\right)} = \frac{3}{2} \xrightarrow{\text{توان } 3} \frac{4}{25}a^2 \times \frac{27}{125}a^3 = \frac{27}{8}$

$\dfrac{4a^5}{25 \times 125} = \dfrac{1}{8} \to a^5 = \dfrac{25 \times 125}{4 \times 8} = \dfrac{5^5}{2^5} \to \boxed{a = \frac{5}{2} = 2.5}$

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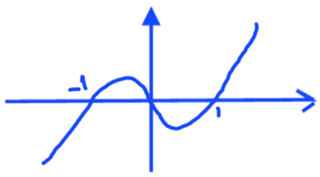
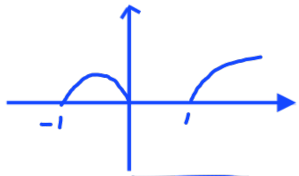
(۸) $y = |x|x - x \to \begin{cases} x^2 - x & x \ge 0 \quad (I) \\ -x^2 - x & x < 0 \quad (II) \end{cases}$




$y = \sqrt{x|x| - x}$

(نقطه بحرانی) $\boxed{K = 4}$ ، (نقطه max) $\boxed{m = 1}$ ، (نقطه min) $\boxed{n = 0}$

$\dfrac{Km + n}{K - n} = \dfrac{4 \times 1 - 0}{4 - 0} = \dfrac{4}{4} = 1$

(۹) $y = \dfrac{mx + 2}{x - 1 + m} \to y' = \dfrac{m(m-1) - 2}{(x - 1 + m)^2} = \dfrac{m^2 - m - 2}{(x - 1 + m)^2} \le 0 \to m^2 - m - 2 \le 0 \to -1 \le m \le 2$

$1 - m \le 1 \to 0 \le m \Rightarrow 0 \le m \le 2 \to \boxed{m = 1}, \boxed{m = 0}$

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(۱۰) $D_{f(x)} = 1 - x|x| = 0 \to x|x| = 1 \to \begin{cases} x \ge 0 & x^2 = 1 \to x = 1 \ ✓ \\ x < 0 & -x^2 = 1 \to x^2 = -1 \ ✗ \end{cases} \to D_f = \mathbb{R} - \{1\}$

مشتق تابع $\begin{cases} x \ge 0 \to f'(x) = \dfrac{1 - x^2 + 2x^2}{(1 - x^2)^2} = \dfrac{x^2 + 1}{(1 - x^2)^2} \to x^2 = -1 \ ✗ \\ x < 0 \to f'(x) = \dfrac{1 + x^2 - 2x^2}{(1 + x^2)^2} = \dfrac{1 - x^2}{(1 + x^2)^2} \to x^2 = 1 \to x = -1 \ ✓ \end{cases}$ → یک نقطه بحرانی