

$$f(x) = 1 - \frac{a}{x} \rightarrow f'(x) = \frac{a}{x^2} \rightarrow \text{آمنده ای}$$

$$f(1) = 1 - a$$

$$f(2) = 1 - \frac{a}{2}$$

$$\frac{a}{2} = \frac{a}{2^2} \rightarrow x = \pm \sqrt{2}$$

$$\text{آمنده متوسط تغییرات} = \frac{1 - \frac{a}{2} - 1 + a}{2} = \frac{a}{2}$$

۱

$$y = 2ax^2 - 2x + 1/a \rightarrow y' = 4ax - 2 \xrightarrow{x=A} 4aA - 2 = 1 \rightarrow A = \frac{3}{4a}$$

چون در نقطه $x=A$ بر نمودار تابع سهم مناسب است یعنی شیب $\neq 0$ و $A \neq 0$

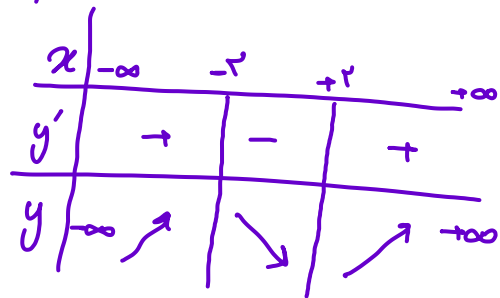
$$\rightarrow A = 2aA^2 - 2A + 1/a \rightarrow \frac{3}{4a} = \frac{9}{4a} - \frac{1}{2} + \frac{1}{4a} \rightarrow 9 = 34a^2 \rightarrow a^2 = \frac{9}{34} \rightarrow a = \pm \frac{3}{\sqrt{34}}$$

۲

$$y = x^3 - 12x^2 + 2 \rightarrow y' = 3x^2 - 24 \Rightarrow 3(x-2)(x+2)$$

$$f(2) = 1 - 24 + 2 = -14$$

$$\min \begin{matrix} 2 \\ -14 \end{matrix}$$



۳

$$y = x^3 + ax^2 - 2bx - 5 \rightarrow y' = 3x^2 + 2ax - 2b \rightarrow b = 0$$

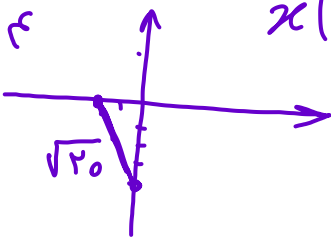
$$y = x^3 + 3x^2 - 5$$

$$x(3x + 2a) = 0 \rightarrow x = 0 \quad |a=3|$$

$$\rightarrow x = -2 \rightarrow -4 + 2a = 0$$

$$f(0) = -5$$

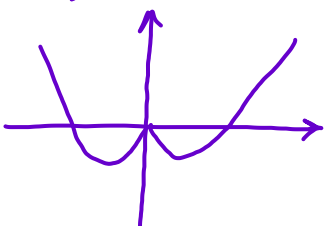
$$f(-2) = 0$$



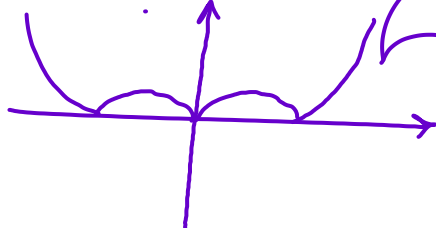
$$d = \sqrt{14 + 5} = \sqrt{19} = \sqrt{20}$$

۴

$$f(x) = x^2 - a|x|$$



$$|f(x)| = |x^2 - a|x||$$



$$\min_{\text{نمی}}(f) = n$$

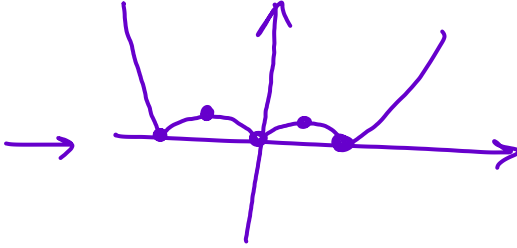
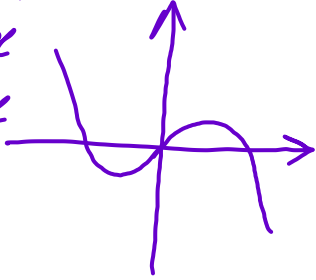
$$\max_{\text{نمی}}(f) = m$$

$$\frac{n}{m} = \frac{2}{3} = \frac{1}{\sqrt{2}}$$

۵

$$f(x) = x(|x| + \nu)$$

$$\begin{aligned} \xrightarrow{+} & x^r + \nu x \\ \xrightarrow{-} & -x^r + \nu x \end{aligned}$$



نقطه بحرانی

۶

$$f(x) = \sqrt[r]{x^r |x-a|} \xrightarrow{a > x > 0} -\sqrt[r]{x^r} (x-a) \quad x(-\nu x + \nu a - \nu)$$

$$f'(x) = \frac{-\nu x^{\nu-1} (x-a)}{\sqrt[r]{x^{\nu r}}} - \sqrt[r]{x^r} = \frac{-\nu x^{\nu-1} + \nu a x^{\nu-1} - \nu x^{\nu}}{\sqrt[r]{x^{\nu r}}} \xrightarrow{x=0} \frac{-\nu x^{\nu-1} + (\nu a - \nu)x^{\nu-1}}{\sqrt[r]{x^{\nu r}}} = 0$$

$$\begin{aligned} -\nu x + \nu a - \nu &= 1, a \\ -\nu + \nu a - \nu &= 1, a \\ \nu a &= \nu + 1, a \end{aligned}$$

$$a = \frac{\nu + 1}{\nu}$$

x	0	1/a
y'	+	-
y	↗	↘

max مطلق

۷

$$f(x) = \sqrt{x|x|} - x \rightarrow x(1 - |x|) \geq 0 \quad \begin{matrix} - & 0 & + \\ + & - & + \end{matrix} \quad D_f = (-\infty, -1] \cup [0, 1]$$

$$f(x) \begin{cases} \rightarrow \sqrt{-x^2+x} & [0, 1] \\ \rightarrow \sqrt{x^2+x} & (-\infty, -1] \end{cases}$$

$$f'(x) \begin{cases} \rightarrow \frac{-2x+1}{2\sqrt{-x^2+x}} & 1 > x > 0 \rightarrow x = \frac{1}{2} \\ \rightarrow \frac{2x+1}{2\sqrt{x^2+x}} & -1 > x \rightarrow \text{ریشه ندارد، نقطه بحرانی} \end{cases}$$

x	0	1/2	1
y'	+	-	+
y	↗	↘	↗

$$\frac{k m + n}{k - n} = \frac{\nu + 0}{\nu - 0} = 1$$

max نسبی $\left(\frac{1}{2}\right)$

۸

$$y = \frac{m x + \nu}{x - 1 + m}$$

$m \neq 1$
نزولی $\left(1, +\infty\right)$

۹

$$f(x) = \frac{x}{1 - x|x|} \begin{cases} \xrightarrow{x > 0} \frac{x}{1 - x^r} \rightarrow \frac{1 - x^r + \nu x^r}{(1 - x^r)^r} \rightarrow \frac{1 + x^r}{(1 - x^r)^r} \rightarrow x = \pm 1 = \text{نقطه بحرانی} \\ \xrightarrow{x < 0} \frac{x}{1 + x^r} \rightarrow \frac{1 + x^r - \nu x^r}{(1 + x^r)^r} = \frac{1 - x^r}{(1 + x^r)^r} = 0 \rightarrow x = \pm 1 \end{cases}$$

۱۰