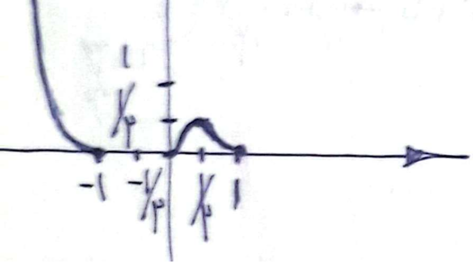
نرگس رزمندل

(1

$f(x)$: $x \geq 0 \Rightarrow \sqrt{x - x^2} = \sqrt{x(1-x)} \Rightarrow [0, 1]$

$x \leq 0 \Rightarrow \sqrt{x + x^2} = \sqrt{x(1+x)} \Rightarrow (-\infty, -1]$



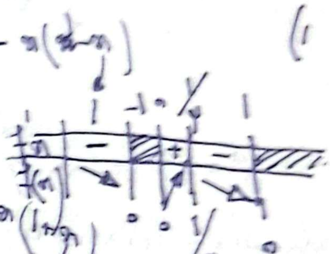
$m = \max$ نسبی $= 1$

$n = \min$ نسبی $= 0$ $\Rightarrow m + n + k = 4$

$k =$ [illegible] $= 3$

$\Rightarrow f'(x)$: $x \geq 0 \Rightarrow \dfrac{1 - 2x}{2\sqrt{x - x^2}} \Rightarrow x = 1/2$, $x(1-x)$

$x \leq 0 \Rightarrow \dfrac{1 + 2x}{2\sqrt{x + x^2}} \Rightarrow x(1+x) \Rightarrow -1$



(۲

$$f'(x) = \frac{1}{2\sqrt{x}} - \frac{2}{2\sqrt{a-2x}} = \frac{\sqrt{a-2x} - 2\sqrt{x}}{4\sqrt{x}\cdot\sqrt{a-2x}} \Rightarrow \sqrt{a-2x} = 2\sqrt{x} \Rightarrow 4x = a-2x \Rightarrow a = 6x$$

$x > 0 \quad a-2x > 0 \Rightarrow 2x < a$

max $f(x) = \sqrt{x} + \sqrt{4x} = \sqrt{4x} + 2\sqrt{x} = 3\sqrt{x}$

$\Rightarrow 3\sqrt{x}\cdot\sqrt{6x} = 3\sqrt{6x^2} = 12 \Rightarrow 6x^2 = 16 \Rightarrow x = +\frac{1}{3}$ ($x > 0$)

$\Rightarrow a = \left[\frac{1}{3} \cdot 6\right] = 14$

min $\Rightarrow \sqrt{a} = \sqrt{6x}$, $x = 0$

(۳

$$f(x) \begin{cases} x \geq 2,\ x \leq -2: & \frac{x^2}{x^2-1}(x^2-4) \\ -2 < x < 2: & \frac{x^2}{x^2-1}(-x^2+5) \end{cases}$$

$x \geq 2,\ x \leq -2$:

$$f'(x) \to \frac{2x(x^2-1) - 2x(x^2)}{(x^2-1)^2}(x^2-5) + (2x)\left(\frac{x^2}{x^2-1}\right)$$

$$= \frac{-2x}{(x^2-1)^2}(x^2-5) + \frac{2x^3}{x^2-1} = \frac{-2x^3 + 8x + 2x^5 - 6x^3}{(x^2-1)^2} = \frac{-4x^3 + 2x^5 + 8x}{(x^2-1)^2}$$

$2x(-4x^2 + x^4 + 5)$: 0 ، ندارد ، ± 1

$-2 < x < 2$:

$$\frac{2x^3 - 8x - 2x^5 + 6x^3}{(x^2-1)^2}$$

	$-\infty$	-1	0	$+1$	
$f'(x)$	$-$	$+$	$-$	$+$	
$f(x)$	↘	↗	↘	↗	$\Rightarrow$ ۳ نقطه

-۴

$y' = 3ax^2 + 2bx + c$

$x=1 \rightarrow 3a + 2b = 0$

$x=0 \rightarrow 0+0+c = 0 \Rightarrow c = 0$

$x=1 \rightarrow y = a+b+c+d = 1 \Rightarrow 3a + 3b = 3$ ($c = 0$, $d = 0$)

$x=0 \rightarrow 0+0+0+d = 0 \Rightarrow d = 0$

$\Rightarrow b = 3 \quad 3a + 9 = 3 \Rightarrow 3a = -6 \Rightarrow a = -2$

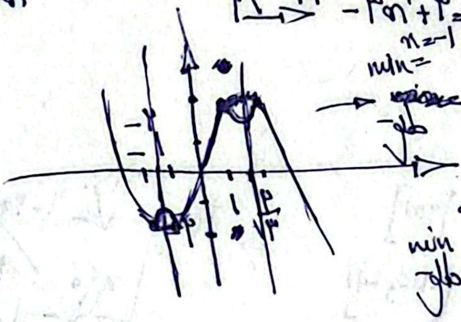
$\Rightarrow ab = +3a - 2 = -6$

-۵

$f(x) = \begin{cases} x < -\sqrt{3},\ x > \sqrt{3} \rightarrow x(-3+x^2) = -3x + x^3 \\ -\sqrt{3} < x < \sqrt{3} \rightarrow x(-x^2+3) = -x^3 + 3x \end{cases}$

$\Rightarrow f'(x) = \begin{cases} x < -\sqrt{3},\ x > \sqrt{3} \rightarrow -3 + 3x^2 = 3(x^2 - 1) \rightarrow \pm 1 \\ -\sqrt{3} < x < \sqrt{3} \rightarrow -3x^2 + 3 = 3(-x^2 + 1) \end{cases}$

	-1		1	
$f'(x)$	$-$	$+$		$-$
$f(x)$	↘	-۲ ↗	+۲	↘



max (x=1) $\Rightarrow \frac{1}{25}\left|3 - \frac{1}{25}\right| = \ldots$

min (x=-1) $\rightarrow -1|3-1| = -2$

-۶

$y' = -3x^2 + 6ax \xrightarrow{x=-1} -3 - 6a = 0 \Rightarrow 6a = -3 \Rightarrow a = -\frac{1}{2}$

$a < 0 \rightarrow y = -x^3 + 3ax^2 + b$

$\Rightarrow -x^3 - \frac{3}{2}x^2 + b \xrightarrow{x=-1} 1 - \frac{3}{2} + b = 1 \rightarrow b = \frac{3}{2}$

$\Rightarrow \frac{b}{a} = \frac{3/2}{-1/2} = -3$

-۷

$y' \Rightarrow 3x + 1 = 0 \Rightarrow x = -\frac{1}{3} \Rightarrow \frac{a}{a+1} = \frac{2}{3} \Rightarrow a = 2$

$x = -\frac{1}{3} \rightarrow y = \frac{3}{2}\left(\frac{1}{9}\right) - \frac{1}{3} + \frac{a}{6} = \frac{2}{3}$

$\Rightarrow y = \frac{2x + 2}{3x+1} = 0 \Rightarrow x = -\frac{2}{3}$

-۸

$f\left(\frac{1}{2}\right) - \frac{1}{2}a + 1 = 0 \Rightarrow 2 - \frac{a}{2} = 0 \Rightarrow a = 4$

$y' = \frac{2bx(4x^2 + 4x + 1) - (8x+4)(bx^2+7)}{4x^2+4x+1} \quad x=1$

$\frac{2b(9) - (12)(b+7)}{9} = 0 \rightarrow 18b - 12b - 7 = 0 \Rightarrow 6b - 7 = 0 \Rightarrow b = \frac{7}{6}$

$x=2 \rightarrow \frac{4b(16+9) - (20)(4b+7)}{25} = 0 \rightarrow 64b + 246 - 80b - 140 = 0$

$\Rightarrow -16b - 104 = 0 \Rightarrow \frac{104}{-16} = b$

$\frac{bx^2 + 7}{(2x+1)^2} = 3$

$12x^2 + 12x + 3 = bx^2 + 7$

$\Rightarrow 3(2x+1)^2 - 7 = bx^2$

$(12 - b)x^2 + 12x - 4 = 0$

$144 + 16(12 - b) = 0 \Rightarrow \frac{144}{-16} = 12 - b \quad b = \frac{144}{12 \cdot 16} = \frac{192 - 144}{16} = \frac{48}{16} = 3 \Rightarrow \frac{b}{a} = \frac{3}{4}$

۹-

$$f'(x)=\frac{۴x^{۳}(x^{۳}-۸)-۳x^{۲}(x^{۴})}{(x^{۳}-۸)^{۲}}=\frac{۴x^{۶}-۳۲x^{۳}-۳x^{۶}}{(x^{۳}-۸)^{۲}}=\frac{x^{۳}(x^{۳}-۳۲)}{(x^{۳}-۸)^{۲}}$$

$x^{۳}\to ۰$ ، $x^{۳}-۳۲\to \sqrt[۳]{۳۲}$ ، $x^{۳}-۸\to ۲$

۰		۲*		$\sqrt[۳]{۳۲}$	
+		−		−	+

$\sqrt[۳]{۳۲}-۲=\sqrt[۳]{۳۲}-\sqrt[۳]{۸}$

۱۰-

$$f'(x)=\frac{۴x^{۳}(x^{۲}-۳)-۲x(x^{۴}-۳)}{(x^{۲}-۳)^{۲}}=\frac{۴x^{۵}-۱۲x^{۳}-۲x^{۵}+۶x}{(x^{۲}-۳)^{۲}}=\frac{۲x^{۵}+۱۲x^{۳}+۶x}{(x^{۲}-۳)^{۲}}$$

$۲x(x^{۴}+۶x^{۲}+۳)$ ، $۲x\to ۰$ ، $(x^{۲}-۳)\to \pm\sqrt{۳}$

$\frac{-۶\pm\sqrt{۲۴}}{۲}=-۳\pm\sqrt{۳}$

	$-۳-\sqrt{۳}$*	$-\sqrt{۳}$*	$-۳+\sqrt{۳}$*	۰	$\sqrt{۳}$*	۴
$f'(x)$	−	−	−	−	+	+