[illegible]

1. $f(x)$ $\xrightarrow{x\ge 0}$ $\sqrt{x-x^2} \rightarrow f'(x)=\frac{-2x+1}{2\sqrt{x-x^2}}$ بحرانی در ۰ و ۱

$\xrightarrow{x<0}$ $\sqrt{x^2+x} \rightarrow f'(x)=\frac{2x+1}{2\sqrt{x^2+x}}$ بحرانی $x=-1$

$0 \quad \frac{1}{2}$: $+ \;|\; -$ Max

$-1 \quad 0$: $+ \;|\; -$ بحرانی

$1 \quad \frac{1}{2}$

$1, 2$

$11, 5$

$\Delta = \ldots$

$k=f(x=\pm 1, 0, \frac{1}{2})=\ldots$

2. $f(x)=\sqrt{x}+\sqrt{a-2x} \rightarrow f'(x)=\frac{1}{2\sqrt{x}}+\frac{-2}{2\sqrt{a-2x}}=\frac{1}{2\sqrt{x}}-\frac{1}{\sqrt{a-2x}}=0$

$\frac{\sqrt{a-2x}-2\sqrt{x}}{2\sqrt{x}\sqrt{a-2x}}=0 \rightarrow x=0 \,/\, x=\frac{a}{2} \,/\, x=\frac{a}{6}$

$f(\frac{a}{2})=\underbrace{\sqrt{\frac{a}{2}}}_{Min} \,/\, f(\frac{a}{6})=\underbrace{\sqrt{\frac{3a}{2}}}_{Max} \,,\, f(0)=\sqrt{a}$

$\sqrt{\frac{a}{2}}\times\sqrt{\frac{3a}{2}}=\sqrt{\frac{3a^2}{4}} \rightarrow \sqrt{\frac{3a^2}{4}}=\sqrt{12} \rightarrow a^2=16 \rightarrow a=4$

$[a]=4$

3. عامل در x ، $a>0$

$x\ge 2 \,-\, x\le -2$: $\frac{x^2}{x^2-1}(x^2-4)=\frac{x^4-4x^2}{x^2-1}$ I $\;f'(x)=\frac{2x(x^4-2x^2+4)}{(x^2-1)^2}=0 \quad \pm 1$

$-2<x<2$: $\frac{x^2}{x^2-1}(4-x^2)=\frac{4x^2-x^4}{x^2-1}$ II $\;f'(x)=\frac{-2x(x^4-2x^2+4)}{(x^2-1)^2} \quad \pm 1$

بحرانی Max

0 : $+ \;|\; -$

$x=2$: $f(2^+)=0 \quad f'_+(2)=+48$; $f(2^-)=0 \quad f'_-(2)=-48$ $\quad \underbrace{x=2}_{مینیمم}$

مینیمم $\max = 0$

مینیمم $\min = \pm 2$

$x=-2$: $f(-2^+)=0 \quad f'_+(-2)=+48$; $f(-2^-)=0 \quad f'_-(-2)=-48$ مینیمم min

PILLNOTEBOOK

آیلا فیروزنیا

۴.

$f'(x) = 3ax^2 + 2bx + c$

$f = ax^3 + bx^2 + cx + d \xrightarrow{x=0} d=0$

$\xrightarrow{x=1} a+b+c=1$

$3ax^2+2bx+c \xrightarrow{x=0} c=0$

$\xrightarrow{x=1} 3a+2b=0$

$c=0 \rightarrow a+b=1 \rightarrow 1=b+a \rightarrow \begin{cases} a+b=1 \\ 3a+2b=0 \end{cases} \xrightarrow{\times -3} \begin{cases} -3a-3b=-3 \\ 3a+2b=0 \end{cases}$

$-b=-3 \rightarrow b=3 \;/\; a=-2$

$ab = 3\times -2 = -6$

۵.

$x \leq -\sqrt{3} \;/\; x \geq \sqrt{3} \rightarrow x^3-3x$

$-\sqrt{3} < x \leq \sqrt{3} \rightarrow 3x - x^3 \rightarrow f'(x) = 3-3x^2 \rightarrow 3(1-x^2)=0$, ± 1

	-1		$+1$	
$-$		$+$		$-$

Min نسبی

$\xrightarrow{x=-1} 3(-1)-(-1)^3 = -2$

$(-1,-2)$

۶.

$-x^3+3ax^2+b \rightarrow y' = -3x^2+6ax$

$-3-6a=0 \rightarrow 6a=-3 \rightarrow a=\frac{-1}{2}$

$-x^3 - \frac{3}{2}x^2 + b \xrightarrow{x=-1} 1-\frac{3}{2}+b=1 \rightarrow b=\frac{3}{2}$

$\frac{b}{a} = \frac{\frac{3}{2}}{-\frac{1}{2}} = -3$

۷. $ext = \left(\frac{-b}{2a}, \frac{-\Delta}{4a}\right) \rightarrow \frac{-b}{2a} = \frac{-1}{3} \;\Big|\; \frac{-\Delta}{4a} = \frac{2}{3}$

جذب قائم — جذب افقی

$\frac{a}{a+1} = \frac{2}{3} \rightarrow 3a = 2a + 2 \rightarrow a = 2$

$\frac{2x+3}{3x+1} = 0 \rightarrow 2x + 3 = 0 \rightarrow \boxed{x = -\frac{3}{2}} \rightarrow \left(-\frac{3}{2}, 0\right)$

۸. $A = \left(\frac{-1}{2}, 3\right)$ $\quad 4\left(\frac{-1}{2}\right)^2 + a\left(\frac{-1}{2}\right) + 1 = 0 \rightarrow \frac{-a}{2} = -2 \rightarrow a = 4$

جذب افقی — جذب قائم

$\frac{b}{4} = 3 \rightarrow b = 12 \qquad \frac{b}{a} = \frac{12}{4} = 3$

۹. $f(x) = \frac{x^4}{x^3-8} \rightarrow f'(x) = \frac{4x^3(x^3-8) - (3x^2)(x^4)}{(x^3-8)^2}$

$= \frac{4x^6 - 32x^3 - 3x^6}{(x^3-8)^2} = \frac{x^6 - 32x^3}{(x^3-8)^2} = \frac{x^3(x^3-32)}{(x^3-8)^2} \qquad \sqrt[3]{32} = 2\sqrt[3]{4}$

$0 \qquad 2 \qquad$ (پایین صورت)

0		2^*		$2\sqrt[3]{4}$	
+	|	−	|	−	| +

$\underbrace{\searrow \quad \searrow}_{\text{اکیدا نزولی}}$

$2\sqrt[3]{4} - 2$ ← کوتاه‌ترین بازه

۱۰. $f(x) = \frac{x^4-3}{x^2-3} \rightarrow f'(x) = \frac{(4x^3)(x^2-3) - (2x)(x^4-3)}{(x^2-3)^2}$

$= \frac{4x^5 - 12x^3 - 2x^5 + 6x}{(x^2-3)^2} = \frac{2x^5 - 12x^3 + 6x}{(x^2-3)^2} = \frac{2x(x^4 - 6x^2 + 3)}{(x^2-3)^2}$

$\frac{6 \pm \sqrt{36-12}}{2} \rightarrow \frac{6+2\sqrt{6}}{2}, \quad \frac{6-2\sqrt{6}}{2}$

$x^2 \simeq 5/4 \rightarrow x \simeq \sqrt{5/4} \qquad x^2 \simeq 0/5 \rightarrow x \simeq \sqrt{0/5} \qquad \pm\sqrt{3}$

$-\sqrt{5/4}$	$-\sqrt{3}^*$	$-\sqrt{0/5}$	0	$\sqrt{0/5}$	$\sqrt{3}^*$	$\sqrt{5/4}$
− | +	| +	| −	| +	| −	| −	| +

۳ بازه صعودی اکید