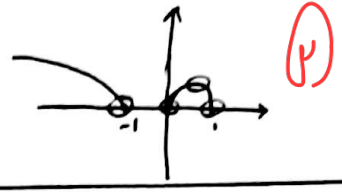


$f(x) = \sqrt{x(1-x)}$   $\rightarrow D_f: x(1-x) \geq 0$   $\rightarrow \begin{matrix} x_1 & - & 1 \\ | & & | \\ | & + & - \\ | & - & + \\ | & + & - \end{matrix}$   $\Rightarrow D_f = (-\infty, -1] \cup [0, 1]$

$x \rightarrow -\infty \Rightarrow y \rightarrow +\infty$

$m=1, n=0, k=4$   
 $\Rightarrow m+n+k=1+0+4=5$  ✓



۱

$D_f: \begin{cases} x \geq 0 \\ a-2x \geq 0 \end{cases} \Rightarrow D_f = [0, \frac{a}{2}]$   $f'(x) = \frac{1}{2\sqrt{x}} + \frac{-1}{\sqrt{a-2x}} = 0 \Rightarrow \frac{1}{2\sqrt{x}} = \frac{1}{\sqrt{a-2x}}$   $\xrightarrow{\text{مربع کردن}}$   $x = a-2x \Rightarrow a = \frac{a}{2}$

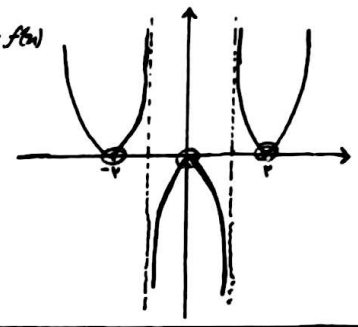
$c \mid \frac{a}{2} \rightarrow y_c = \sqrt{\frac{a}{2}} + \sqrt{\frac{2a}{2}} = \sqrt{\frac{a}{2}} + \sqrt{a} = 3\sqrt{\frac{a}{2}} = \sqrt{\frac{9a}{2}} = \sqrt{\frac{9a}{2}}$  (2)

$y_{max} \cdot y_{min} = \sqrt{12} \rightarrow \sqrt{\frac{9a}{2}} \cdot \sqrt{\frac{a}{2}} = \sqrt{12} \Rightarrow a = [a] = 4$  ✓

۲

$f(x) = \frac{x^2}{x^2-1} |x^2-1| \rightarrow$  صحت  $\begin{cases} x=0 \rightarrow$  مضاعف ✓  $x = \pm 1 \rightarrow$  تقاطع ساده  $x = \pm 1 \rightarrow$  مخرج = 0  $\Rightarrow x = \pm 1$   $\rightarrow$  جانب تعام انتقال ساده

$\lim_{x \rightarrow \pm\infty} f(x) \xrightarrow{\text{تقریب}} \frac{x^2}{a^2} |x^2| \rightarrow +\infty$   $\rightarrow$  نامعین انتی /  $A(0,0) \in f(x)$  /  $f$  تابعی زوج



۳

$y = ax^3 + bx^2 + cx + d \rightarrow y' = 3ax^2 + 2bx + c$

$A(0,0) \rightarrow \begin{cases} a(0)^3 + b(0)^2 + c(0) + d = 0 \Rightarrow d = 0 \\ 3a(0)^2 + 2b(0) + c = 0 \Rightarrow c = 0 \end{cases} \Rightarrow \begin{cases} y = ax^3 + bx^2 \\ y' = 3ax^2 + 2bx \end{cases} / B(1,1) \rightarrow \begin{cases} a(1)^3 + b(1)^2 = 1 \Rightarrow a+b=1 \\ 3a(1) + 2b(1) = 0 \Rightarrow 3a+2b=0 \end{cases}$

$\Rightarrow \begin{cases} 3a+2b=0 \\ a+b=1 \end{cases} \rightarrow a = -2, b = 3 \quad | \quad ab = -6$  ✓ (2)

۴

$f(x) = x|x-2| \quad [-1, \sqrt{3}] \xrightarrow{\text{مربع کردن}} f(x) = -x^2 + 2x \rightarrow f'(x) = -2x + 2 = 0$  (2)

$f(-1) = -9/8 \quad f(\sqrt{3}) = 0 \Rightarrow \begin{cases} A(1, 2) \\ B(-1, -2) \end{cases} \rightarrow y_{min} = -2$   $\rightarrow$   $y_{min} = -2$   $\rightarrow$   $y_{min} = -2$   $\rightarrow$   $y_{min} = -2$

۵

$$y = x^2|x| + 3ax^2 + b \xrightarrow{x=-1} y = -x^3 + 3ax^2 + b \rightarrow y' = -3x^2 + 6ax \xrightarrow{x=-1} -3 - 6a = 0 \Rightarrow a = -\frac{1}{2}$$

$$\frac{b}{a} = \frac{-\frac{1}{2}}{-\frac{1}{2}} = 1 \Rightarrow b = 1$$

$$\xrightarrow{x=1} 1 + 3a + b = 1 \xrightarrow{a=-\frac{1}{2}} b = \frac{3}{2} \checkmark$$

4

$$(a+1)x + (a-1) = 0 \Rightarrow x = \frac{1-a}{a+1} \rightarrow \text{مجاوب قائم}$$

$$\frac{a}{a+1} = y \rightarrow \text{مجاوب افقی}$$

$$\left. \begin{array}{l} \\ \end{array} \right\} \text{محل تقاطع: } A\left(\frac{1-a}{a+1}, \frac{a}{a+1}\right)$$

5

$$y = \frac{3}{4}x^2 + x + \frac{1}{4} \rightarrow A\left(-\frac{1}{4}, \frac{3}{4}\right) \Rightarrow \frac{-a+1}{a+1} = -\frac{1}{3} \Rightarrow \frac{a-1}{a+1} = \frac{1}{3} \Rightarrow 3a - 3 = a + 1 \Rightarrow a = 2$$

$$y = \frac{3a+3}{3a+1} = 0 \Rightarrow a = -\frac{3}{2}$$

✓

5

$$y = \frac{bx^2 + 7}{4x^2 + ax + 1} \quad \text{مجاوب قائم} \rightarrow x = -\frac{1}{4} \Rightarrow 4\left(-\frac{1}{4}\right)^2 + a\left(-\frac{1}{4}\right) + 1 = 0 \rightarrow \frac{a}{4} = 2 \Rightarrow a = 8$$

6

$$\text{مجاوب افقی} \rightarrow y = 3 \Rightarrow \lim_{x \rightarrow \pm\infty} y = \frac{b}{4} = 3 \Rightarrow b = 12$$

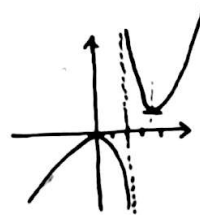
$$\frac{b}{a} = \frac{12}{8} = \frac{3}{2}$$

8

$$f(x) = \frac{x^2}{x^3 - 1}$$

$x^2 = 0 \rightarrow$  مجاوب:  $x = 0$

$x^3 - 1 = 0 \Rightarrow x = 1 \rightarrow$  مجاوب قائم  
انتقال سهم



7

$$\lim_{x \rightarrow +\infty} f(x) = +\infty$$

$$\lim_{x \rightarrow -\infty} f(x) = -\infty$$

$$f'(x) = \frac{x^2 - 3x^3}{(x^3 - 1)^2} = 0 \Rightarrow x^2(x^2 - 3) = 0 \Rightarrow \begin{cases} x = 0 \\ x = \sqrt{3} \rightarrow 3 < \sqrt{3} < 4 \\ x = -\sqrt{3} \rightarrow 3 < \sqrt{3} < 4 \end{cases}$$

$$\hookrightarrow = 3, 17$$

بانهایی

$$\text{دانهایی که در آن کجای نزولی است} : (0, 2) \cup (2, \sqrt{3}) \Rightarrow \text{مستقیم طول} = \sqrt{3} - 2$$

از این اندازه

✓

9

$$f(x) = \frac{x^2 - 3}{x^2 - 3}$$

$$x^2 - 3 = 0 \Rightarrow x = \pm\sqrt{3} \text{ (مستقیم)}$$

$$x^2 - 3 = 0 \Rightarrow x = \pm\sqrt{3} \text{ (مجاوب قائم)}$$

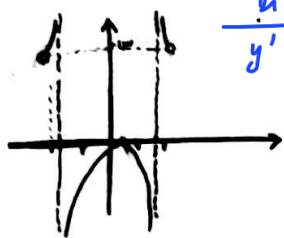
$$f(-\sqrt{3}) = 13$$

$$f(\sqrt{3}) = 13$$

- تابع f، زوج -

8

$$\lim_{x \rightarrow \pm\infty} f(x) = +\infty$$



$x_1$	$-\sqrt{3}$	$-\sqrt{2+3}$	$0$	$\sqrt{2+3}$	$\sqrt{3}$
$y'$	$-$	$-$	$+$	$-$	$+$

بازدهایی که در آن کجای نزولی است  
 $(-\sqrt{3}, \sqrt{3}) \cup (\sqrt{3}, 2)$

$\Rightarrow$  بازه 2 -

$$12a^2 - 12a^3 + 4a = 0 \rightarrow 12a^2(a^2 - a + \frac{1}{3}) = 0 \rightarrow \{a = 0\}$$

$$\rightarrow a^2 - a + \frac{1}{3} = 0 \xrightarrow{a^2 = t} t^2 - 4t + 4 = 0 \rightarrow t = \frac{4 \pm \sqrt{16 - 4}}{2} = 2 \pm \sqrt{2} \rightarrow \begin{cases} x = \pm \sqrt{2 + \sqrt{2}} \\ x = \pm \sqrt{2 - \sqrt{2}} \end{cases}$$

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