۱)

$f(x) = \begin{cases} \sqrt{x-x^2} & x \geq 0 \\ \sqrt{x+x^2} & x < 0 \end{cases} \rightarrow f'(x) = \begin{cases} \sqrt{1-2x} & x \geq 0 \\ \sqrt{1+2x} & x < 0 \end{cases}$

$x \geq 0 \rightarrow \sqrt{1-2x} = 0 \rightarrow x = \frac{1}{2}$ ✓ نقطه بحرانی

$x < 0 \rightarrow \sqrt{1+2x} = 0 \rightarrow x = -\frac{1}{2}$ X در دامنه نیست

$D_f = (-\infty, -1] \cup [0, 1]$

	0	$\frac{1}{2}$	1
y'	+	−	
y	↗	↘	

$\rightarrow \frac{1}{2}$: max نسبی $m = 1$, $n = 0$

$\rightarrow \{0, \pm 1, \frac{1}{2}\} \rightarrow$ ۴ عضو $\rightarrow k = 4 \rightarrow m + n + k = 5$

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۲)

$f'(x) = \frac{1}{2\sqrt{x}} + \frac{-1}{\sqrt{a-2x}} = 0 \rightarrow \frac{\sqrt{a-2x} - 2\sqrt{x}}{2\sqrt{x}\sqrt{a-2x}} = 0 \rightarrow x = 0$ یا $x = \frac{a}{2}$ یا $x = \frac{a}{6}$

$\underbrace{f(\frac{a}{2}) = \sqrt{\frac{a}{2}}}_{min}$, $\underbrace{f(\frac{a}{6}) = \sqrt{\frac{3a}{2}}}_{max}$, $f(0) = \sqrt{a}$

$\sqrt{\frac{3a}{2}} \times \sqrt{\frac{a}{2}} = \sqrt{12} \rightarrow \frac{3a^2}{4} = 12$

$a^2 = 16 \rightarrow a = 4$

$[a] = 4$

---

۳)

$\frac{x^2}{x^2-1}|x^2-4| \longrightarrow \frac{x^4-4x^2}{x^2-1} \longrightarrow \frac{(4x^3-8x)(x^2-1)-(2x)(x^4-4x^2)}{(x^2-1)^2} = 0 \longrightarrow$

$\frac{2x^5-4x^3+8x}{(x^2-1)^2} = 0 \rightarrow 2x\underbrace{(x^4-2x^2+4)}_{\text{ریشه ندارد}} = 0 \rightarrow x = 0$

* ریشه داخل قدر مطلق با توان ۱ هم اکسترمم است

ext : $\{-2, 0, 2\} \longrightarrow$ ۳ نقطه

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۴)

$3ax^2 + 2bx + c \xrightarrow{(0,0)} c = 0$

$\xrightarrow{(1,0)} 3a + 2b = 0$

$ax^3 + bx^2 + cx + d \xrightarrow{(0,0)} d = 0$

$\xrightarrow{(1,1)} a + b = 1 \rightarrow$

$a = -2$, $b = 3 \longrightarrow \boxed{ab = -6}$

---

۵)

$f(x) = 3x - x^3 \longrightarrow f'(x) = 3 - 3x^2 = 0 \longrightarrow x = \pm 1$

	$-\frac{3}{2}$	-1	1	$\sqrt{3}$
y'	−	+	−	
y	↘	↗	↘	

$f(-\frac{3}{2}) = -\frac{9}{8}$ $\underbrace{f(-1) = -2}_{\text{min مطلق}}$ $f(1) = 2$ $f(\sqrt{3}) = 0$

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۶)

$y = -x^3 + 3ax^2 + b \rightarrow y' = -3x^2 + 6ax \xrightarrow{(-1,0)} -3 - 6a = 0 \longrightarrow a = -\frac{1}{2}$

$(-1, 1) : 1 + 3a + b = 1 \longrightarrow b = \frac{3}{2} \longrightarrow \frac{b}{a} = -3$

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۷)

min تابع : $(\frac{-b}{2a}, \frac{-\Delta}{4a}) \longrightarrow S(-\frac{1}{3}, \frac{2}{3})$

$\frac{a}{a+1} = \frac{2}{3} \longrightarrow a = 2 \longrightarrow \frac{2x+3}{3x+1} = 0 \longrightarrow x = \frac{-3}{2}$

Scanned with CamScanner

(۸

$A(-\frac{1}{2}, 3)$

$4(-\frac{1}{2})^2 + a(-\frac{1}{2}) + 1 = 0 \longrightarrow -\frac{a}{2} = -2 \longrightarrow a = 4$

$\frac{b}{4} = 3 \longrightarrow b = 12 \longrightarrow \frac{b}{a} = 3$

(۹

$$f'(x) = \frac{(x^3-8)(4x^3) - (3x^2)(x^4)}{(x^3-8)^2} = \frac{x^3(x^3-32)}{(x^3-8)^2} \lessgtr 0 \longrightarrow$$

		0		2		$2\sqrt[3]{4}$	
y'	$+$		$-$		$-$		$+$
y	↗		↘		↘		↗

$\longrightarrow$ طول کوتاه‌ترین بازه : $2\sqrt[3]{4} - 2$

(۱۰

$$f'(x) = \frac{4x^3(x^2-3) - 2x(x^4-3)}{(x^2-3)^2} = \frac{4x^5 - 12x^3 - 2x^5 + 6x}{(x^2-3)^2} = 0$$

		-2		$-\sqrt{3}$		$-\sqrt{3-\sqrt{6}}$		0		$\sqrt{3-\sqrt{6}}$		$\sqrt{3}$		2	
y'			$-$		$-$		$+$		$-$		$+$		$+$		
y			↘		↘		↗		↘		↗		↗		

$\longrightarrow$ اکیداً نزولی در ۳ بازه