(۲)

$$f(x)=\sqrt{x}+\sqrt{a-2x} \Rightarrow \begin{cases} x\geq 0 \\ x\leq \frac{a}{2} \end{cases} \Rightarrow Df=[0,\frac{a}{2}]$$

$$f'(x)=\frac{1}{2\sqrt{x}}-\frac{1}{\sqrt{a-2x}}=0 \Rightarrow \frac{1}{2\sqrt{x}}=\frac{1}{\sqrt{a-2x}} \Rightarrow 4x=a-2x \Rightarrow x=\frac{a}{6}$$

$$\begin{cases} f(0)=\sqrt{a} \\ f(\frac{a}{6})=\sqrt{\frac{a}{6}}+\sqrt{\frac{4a}{6}}=3\sqrt{\frac{a}{6}} \Rightarrow y_{min}=\sqrt{\frac{a}{2}} \\ f(\frac{a}{2})=\sqrt{\frac{a}{2}} \end{cases} \quad y_{max}=3\sqrt{\frac{a}{6}} \Big\} x=\sqrt{12}$$

$$\Rightarrow \frac{3a}{12}=\sqrt{2}$$

$$\Rightarrow$$

$$a=4$$

$$\Rightarrow [a]=4$$

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(۳)

$$g(x)=\frac{x^2(x^2-4)}{x^2-1}=\frac{x^4-4x^2}{x^2-1}$$

$$g'(x)=\frac{(4x^3-8x)(x^2-1)-2x(x^4-4x^2)}{(x^2-1)^2}=\frac{2x((2x^2-4)(x^2-1)-x^4+4x^2)}{(x^2-1)^2}$$

$$=\frac{2x(x^4-2x^2+4)}{(x^2-1)^2}$$

$$f(x)=\begin{cases} g(x) \to |x|>2 \\ -g(x) \to |x|<2 \end{cases} \Rightarrow f'(x)=\begin{cases} g'(x) \to |x|>2 \\ -g'(x) \to |x|<2 \end{cases}$$

$\Leftarrow f$ در ۳ نقطه $\leftarrow x=2, x=-2, x=0$ تقسیم $\Rightarrow$ اکسترمم!

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(۴)

$$f(0)=0 \Rightarrow d=0 \Rightarrow f(1)=a+b+c=1$$

$$f'(0)=f'(1)=0$$

$$f'(x)=3ax^2+2bx+c \Rightarrow \begin{cases} f'(0)=c=0 \\ f'(1)=3a+2b=0 \end{cases} \Rightarrow \begin{cases} a+b=1 \\ 3a+2b=0 \end{cases}$$

$$a=-2$$

$$b=3$$

$$\Rightarrow a+b=-4$$

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۶) $3-x^2=0 \Rightarrow x^2=3 \Rightarrow \begin{cases} x=\sqrt{3} & \text{قبول} \\ x=-\sqrt{3} & \text{غ} \end{cases}$

$f(x)=x(3-x^2)=3x^2-x^3 \xrightarrow{f'=0} f'(x)=6x-3x^2=0$

$\Rightarrow x^2=1 \Rightarrow x=\pm 1$

$f(-1,0)=-1,0\times|3-2,25|=\frac{-9}{8}$

$f(-1)=-1\times|1^2-1|=-2 \rightarrow$ min مطلق

$f(1)=|1^2-1|=2$

$f(\sqrt{3})=\sqrt{3}\times|1^2-3|=0$

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۷) $x=-1 \rightarrow |x|=-x \Rightarrow y=-x^3+3ax^2+b$

$A(-1,1)$

$\hookleftarrow$ اکستریمی $\Rightarrow f(-1)=1$ ، $f'(-1)=0$

$\Rightarrow 1+3a+b=1 \Rightarrow b=-3a \rightarrow b=\frac{3}{2}$

$y'=-3x^2+6ax \xrightarrow{x=-1} -3-6a=0 \Rightarrow a=\frac{-1}{2}$

$\Rightarrow \frac{b}{a}=\frac{\frac{3}{2}}{\frac{-1}{2}}=\frac{3,2}{-2\times 1}=-3$

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۸) $f'=\frac{4x^3(x^3-1)-3x^2(x^4)}{(x^3-1)^2}=\frac{4x^6-3x^3-3x^6}{(x^3-1)^2}$

$=\frac{x^6-3x^3}{(x^3-1)^2} \xrightarrow{f'=0} x^6-3x^3=0 \Rightarrow \begin{cases} x=0 \\ x=\sqrt[3]{3^3}=2\sqrt[3]{4} \end{cases}$

x	0		2		$2\sqrt[3]{4}$
f'	$+$	0 $-$	‖	$-$	0 $+$
f	↗	↘	‖	↘	↗

نقطه‌ی $x=2$ نقطه‌ی تنوع است

نقطه‌ی $x=0$ نقطه‌ی بازه $\Leftarrow$ نقطه‌ی بازه $(0,2)$ ، $(2,2\sqrt[3]{4})$ ابدا تنوی است

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$Df = R - \{\pm\sqrt{3}\}$ (1.)

$$f'(x) = \frac{(4x^3)(x^2-3)-(2x)(x^4-3)}{(x^2-3)^2} = \frac{4x^5-12x^3-2x^5+6x}{(x^2-3)^2}$$

$$\Rightarrow \frac{2x^5-12x^3+6x}{(x^2-3)^2} = \frac{2x(x^4-6x^2+3)}{(x^2-3)^2} = \frac{2x(x^2-1)(x^2-3)}{(x^2-3)^2}$$

$$= \frac{2x(x^2-1)}{(x^2-3)}$$

$2x = 0 \Rightarrow x = 0$

$x^2-1 = 0 \Rightarrow x = \pm 1$

$x^2-3 = 0 \Rightarrow x = \pm\sqrt{3}$

(۳ نمره)

x	-2	$-\sqrt{3}$	-1	0	1	$\sqrt{3}$	2
$f'(x)$	‖	$-$	0	$+$	0	$-$	0	$+$	0	$-$	0	$+$	‖
$f(x)$	‖	↘	min	↗	max	↘	min	↗	max	↘	min	↗	‖

(ii)

جانشین $\rightarrow (a+1)x + (a-1) = 0 \Rightarrow (a+1)x = -(a-1) \Rightarrow x = \frac{-(a-1)}{a+1}$

جانب اول $\rightarrow y = \frac{a}{a+1}$ شامل $x = \frac{-b}{2a} \Rightarrow \frac{-1}{2\times\frac{3}{2}} = -\frac{1}{3}$ نقطه min

$$\frac{1}{3} = \frac{-(a-1)}{a+1} \Rightarrow a+1 = 3a-3 \Rightarrow 2a = 4 \Rightarrow a = 2 \longrightarrow y = \frac{2x+3}{3x+1}$$

نقطه برخورد با محور xها $\rightarrow y = 0 \rightarrow$ صورت $= 0 \Rightarrow 2x+3 = 0 \Rightarrow x = \frac{-3}{2}$