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ریاضیات گسسته ~

۱- تعداد عبارات درست

m ← بیشترین ماکس
n ← بیشترین مین
k ← تعداد عبارات درست

k + m + n

$f(x) = \sqrt{x(1-|x|)}$

$x = 0$

$f(x) = \sqrt{x+x^2}$ | $f(x) = \sqrt{x-x^2}$ $D_f : [0,1]$

$D_f : (-\infty,-1] \cup [0,+\infty)$ | $f'(x) = \frac{1-2x}{2\sqrt{x-x^2}} = 0 \to x = \frac{1}{2}$ ✓

$f'(x) = \frac{1+2x}{2\sqrt{x^2+x}} = 0 \to x = -\frac{1}{2}$ ✗ | $x = 0$ ✓ , $x = 1$ ✓

$x(x+1) = 0$

$x = 0$ ✓ , $x = -1$ ✓

مقادیر عبارات تابع: $\{-1, 0, 1, \frac{1}{2}\}$

$k = 4$

$m = 1$

$n = 0$

$k + m + n = 5$

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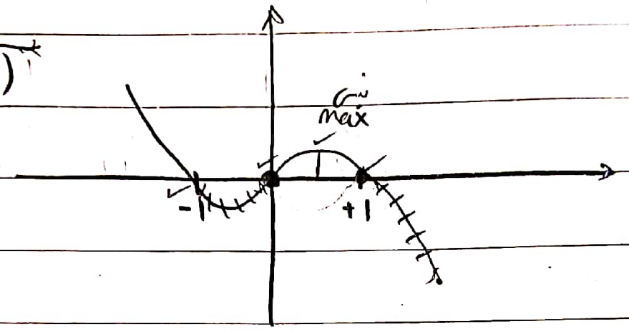
$\sqrt{x(1-|x|)}$



۲- $f(x) = \sqrt{x} + \sqrt{a-2x}$ $\quad \max_{\text{مطلق}} \times \min_{\text{مطلق}} = \sqrt{12}$

$f'(x) = \frac{1}{2\sqrt{x}} + \frac{-2}{2\sqrt{a-2x}} = \frac{\sqrt{a-2x} + 2\sqrt{x}}{2\sqrt{x}\sqrt{a-2x}} = 0$ $\quad a > 0 \to [a] = ?$

$\sqrt{a-2x} = -2\sqrt{x} \Rightarrow a - 2x = 4x \Rightarrow a = 6x \Rightarrow x = \frac{a}{6}$

$2\sqrt{x}\sqrt{a-2x} = 0 \to x = 0$; $2x = a \Rightarrow x = \frac{a}{2}$

۲ ✓ عبارت درست

$x = \frac{a}{6} \to \sqrt{\frac{a}{6}} + \sqrt{a - \frac{a}{3}} = \sqrt{\frac{a}{6}} + \sqrt{\frac{2a}{3}} = \frac{\sqrt{a}}{\sqrt{6}} + \frac{2\sqrt{a}}{\sqrt{6}} = 3\sqrt{\frac{a}{6}} = \frac{\sqrt{3a}}{\sqrt{2}}$ $\quad \max$

$x = 0 \to \sqrt{a} = \sqrt{\frac{3a}{2}}$

$x = \frac{a}{2} \to \sqrt{\frac{a}{2}} + \sqrt{a-a} = \sqrt{\frac{a}{2}} = \sqrt{\frac{3a}{6}}$ $\quad \min$ ✓

$\sqrt{\frac{3a}{6} \times \frac{3a}{6}} = \sqrt{12} \quad \sqrt{\frac{3|a|}{2}} = \sqrt{12} = 2\sqrt{3} \Rightarrow |a| = 8 \xrightarrow{a>0} a = 8$

$[a] = 4$

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$D_f = \mathbb{R} - \{\pm 1\}$

$f(x) = \frac{x^2}{x^2-1}\left|x^2-4\right|$

$x^2 = 4 \rightarrow |x = \pm 2|$ ... -۳

$f(x) = \frac{x^2+0}{x^2-1}(x^2-4) = u.v \rightarrow f'(x) = u'v + v'u$

$f'(x) = \frac{-1}{(x^2-1)^2}(2x)(x^2-4) + (2x)\left(\frac{x^2}{x^2-1}\right)$

$f'(x) = \frac{-2x(x^2-4)+(2x)(x^2)(x^2-1)}{(x^2-1)^2}$

$f'(x) = \frac{2x(-x^2+4+x^4-x^2)}{(x^2-1)^2}$ (۲)

$f'(x) = \frac{2x(x^4-2x^2+4)}{(x^2-1)^2} = 0 \rightarrow \boxed{x=0}$ ✓

$(x^2-1)^2 = 0 \rightarrow x = \pm 1$ ق ق $\notin D_f$

	-2		0		$+2$	
y'	$-$	$+$	0	$-$	0	$+$
y	↘	↗		↘		↗

۳: تعداد اکسترمم‌ها

A(0,0) B(1,1) ab=? -۴

$f(x): y = ax^3 + bx^2 + cx + d$

$A \in f(x) \Rightarrow d = 0$

$B \in f(x) \Rightarrow a + b + \cancel{c} = 1 \Rightarrow a + b = 1$ * (۲)

$f'(x) = 3ax^2 + 2bx + \cancel{c} = 0$

$\rightarrow c = 0$

$\rightarrow 3a + 2b = 0$ *

$\Rightarrow \begin{cases} a = -2 \\ b = 3 \end{cases} \quad ab = \boxed{-6}$ ✓

۵- $x = \pm\sqrt{3}$

$f(x) = x|3 - x^2|$ $\quad x \in [-1.5, \sqrt{3}]$ مطلق min ؟

$f(x) = 3x - x^3$

$f'(x) = 3 - 3x^2 = 3(1 - x^2) = 0 \rightarrow x = \pm 1$

$f(\sqrt{3}) = 0$

$f(-\sqrt{3}) = 0$

$f(1) = 2$

$f(-1) = -2$ min مطلق ✓

$\Rightarrow$ مطلق $\min = -2$ ✓ (۲)

۶-

$A(-1, 1)$ $\quad \frac{b}{a} = ?$

$f(x) = y = x^2|x| + 3ax^2 + b$ $\Rightarrow \frac{b}{a} = -3$

$A \in f(x) \Rightarrow 1 + 3a + b = 1 \Rightarrow 3a + b = 0$ $\quad b = -\frac{3}{2}$

$f(x) = x^3 + 3ax^2 + b \rightarrow f'(x) = 3x^2 + 6ax$

$f'(-1) = 0 \rightarrow 3 - 6a = 0 \rightarrow a = \frac{1}{2}$ (۴) ✓

۷-

$y = \frac{(ax + 3)}{(a+1)x + (a-1)}$

مجانب قائم: $(a+1)x = 1 - a \Rightarrow x = \frac{1-a}{a+1}$

مجانب افقی: $y = \frac{a}{a+1}$

محل تلاقی مجانب‌ها $\left(\frac{1-a}{a+1}, \frac{a}{a+1}\right)$

$y = \frac{3}{2}x^2 + x + \frac{5}{6}$

min

$\min \left| \begin{array}{l} \frac{-1}{3} \\ \frac{3}{2}\left(\frac{1}{9}\right) - \frac{1}{3} + \frac{5}{6} = \frac{1}{6} - \frac{2}{6} + \frac{5}{6} = \frac{2}{3} \end{array} \right.$ (۴) ✓

$\Rightarrow \frac{1-a}{a+1} = \frac{-1}{3} \Rightarrow 3 - 3a = -a - 1 \rightarrow 4a = 4 \rightarrow a = 2$

$\frac{6x + 3}{3x + 1} = 0 \rightarrow x = -\frac{3}{6} = -\frac{1}{2}$

محل تلاقی تابع همواره با محور x ها ؟

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$A\left(-\frac{1}{2}, 2\right)$ $\quad f(x)$ $\quad y = \frac{bx^2 + 7}{4x^2 + ax + 1}$ $\quad \frac{b}{a} = ?$

$\hookrightarrow x = \frac{-1}{2}$ (نقطه عزت) $\quad y = 1^{\circ}$

$+1 - \frac{a}{2} + 1 = 0$

$2 = \frac{a}{2} \Rightarrow \boxed{a = 8}$

$* \lim_{x \to \infty} f(x) = \frac{b}{4} = 2 \Rightarrow \boxed{b = 12}$

$\left\{ \frac{b}{a} = \frac{12}{8} = \frac{3}{2} \right.$ (۲)

۹-

$f(x) = \frac{x^4}{x^3 - 8}$

$f'(x) = \frac{4x^3(x^3 - 8) - (3x^2)(x^4)}{(x^3 - 8)^2}$

$f'(x) = \frac{x^3(4x^3 - 32 - 3x^3)}{(x^3 - 8)^2} = \frac{x^3(x^3 - 32)}{(x^3 - 8)^2}$ $\quad x = \sqrt[3]{32}$ $\quad x^3 = 0$ $\quad x = 2$ (۲)

x	0		2		$\sqrt[3]{32}$	
y'	$+$	0	$-$	$-$	0	$+$
y	↗		↘	↘		↗

تابع اکیدا: $(0, 2) \cup (2, \sqrt[3]{32})$ ؛ طول بازه‌ها: 2 ، $\sqrt[3]{32} - 2$ ، $2 - 0$

۱۰-

$f(x) = \frac{x^4 - 3}{x^2 - 3}$ $\quad x \in (-2, 2)$ $\quad D_f : \mathbb{R} - \{\pm\sqrt{3}\}$

$f'(x) = \frac{(4x^3)(x^2 - 3) - (2x)(x^4 - 3)}{(x^2 - 3)^2}$

$f'(x) = \frac{4x^5 - 12x^3 - 2x^5 + 6x}{(x^2 - 3)^2} = \frac{2x^5 - 12x^3 + 6x}{(x^2 - 3)^2}$

$f'(x) = \frac{2x(x^4 - 6x^2 + 3)}{(x^2 - 3)^2} = 0 \begin{cases} x = 0 \\ x^4 - 6x^2 + 3 = 0 \end{cases}$ (۲)

$\hookrightarrow x = \pm\sqrt{3}$

x	-2	$-\sqrt{3}$	$-\sqrt{3-\sqrt{6}}$	0	$\sqrt{3-\sqrt{6}}$	$\sqrt{3}$	2	
	$-$	$-$	$+$	$-$	$+$	$+$		
	↘	↘	↗	↘	↗	↗		

* در بسیار نزدیک