«کلیت ۲۶»

«دوازدهم رشته‌ی A» «مبینا عباسی»

۱- $\begin{cases} m \leftarrow \max \text{ نسبی} \\ n \leftarrow \min \text{ نسبی} \\ k \leftarrow \text{تعداد بحرانی‌ها} \end{cases}$

$k+m+n$

$$f(x) = \sqrt{x(1-|x|)}$$

$x=0$

$f(x) = \sqrt{x+x^2}$

$D_f: (-\infty,-1] \cup [0, +\infty)$

$f'(x) = \dfrac{1+2x}{2\sqrt{x^2+x}}$; $1+2x = 0 \rightarrow x = \frac{-1}{2}$ ✗

$\hookrightarrow x(x+1)=0$

$x=0$ ✓ $\quad x=-1$ ✓

$f(x) = \sqrt{x-x^2} \quad D_f: [0,1]$

$f'(x) = \dfrac{1-2x}{2\sqrt{x-x^2}}$; $1-2x = 0 \longrightarrow x = \frac{1}{2}$ ✓

$2\sqrt{x-x^2} = 0 \longrightarrow x=0$ ✓, $x=1$ ✓

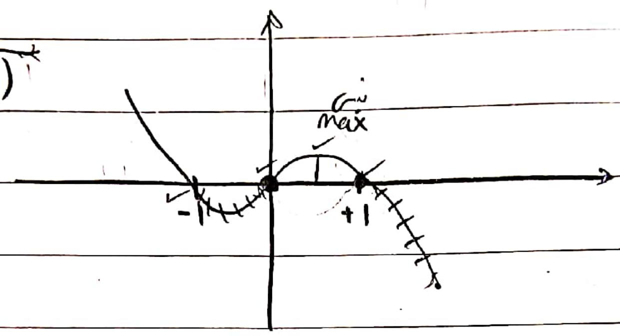
$\left\{\frac{1}{2}, 0, 1, -1\right\}$ نقاط بحرانی تابع

$\boxed{k=4}$

$\boxed{m=1}$

$\boxed{n=0}$

$\sqrt{x(1-|x|)}$



$k+m+n = \underline{\underline{5}}$

۲- $f(x) = \sqrt{x} + \sqrt{a-2x}$ $\quad$ max مطلق $\times$ min مطلق $= \sqrt{12}$ $\quad a>0 \rightarrow [a] = ?$

$$f'(x) = \frac{1}{2\sqrt{x}} + \frac{-\cancel{2}}{\cancel{2}\sqrt{a-2x}} = \frac{\sqrt{a-2x} + 2\sqrt{x}}{2\sqrt{x}\sqrt{a-2x}} = 0$$

$\sqrt{a-2x} = -2\sqrt{x} \Rightarrow a-2x = 4x \Rightarrow a = 6x \Rightarrow x = \frac{a}{6}$ $\quad$ بحرانی‌ها

$2\sqrt{x}\sqrt{a-2x} = 0 \begin{cases} \rightarrow x=0 \\ \rightarrow 2x = a \Rightarrow x = \frac{a}{2} \end{cases}$

$x = \frac{a}{6} \rightarrow \sqrt{\frac{a}{6}} + \sqrt{a - \frac{a}{3}} = \sqrt{\frac{a}{6}} + \sqrt{\frac{2a}{3}} = \frac{\sqrt{a}}{\sqrt{6}} + \frac{2\sqrt{a}}{\sqrt{6}} = 3\sqrt{\frac{a}{6}} = \sqrt{\frac{9a}{6}}$ max

$x=0 \longrightarrow \sqrt{a} = \sqrt{\frac{6a}{6}}$

$x = \frac{a}{2} \rightarrow \sqrt{\frac{a}{2}} + \sqrt{\cancel{a}-\cancel{a}} = \sqrt{\frac{a}{2}} = \sqrt{\frac{3a}{6}}$ min

$\sqrt{\frac{\cancel{9}a}{\cancel{6}} \times \frac{3a}{\cancel{6}}} = \sqrt{12} \qquad \frac{\sqrt{3}\,|a|}{2} = \sqrt{12} = 2\sqrt{3} \Rightarrow |a| = 4 \xrightarrow{a>0} a = 4$

$\boxed{[a] = 4}$

$D_f = \mathbb{R} - \{\pm 1\}$ $\qquad x^2 = 4 \rightarrow \boxed{x = \pm 2}$ -۳

$$f(x) = \frac{x^2}{x^2-1}\left|x^2-4\right|$$

$$f(x) = \frac{x^2+0}{x^2-1}(x^2-4) = u \cdot v \rightarrow f'(x) = u'v + v'u$$

$$f'(x) = \frac{-1}{(x^2-1)^2}(2x)(x^2-4) + (2x)\left(\frac{x^2}{x^2-1}\right)$$

$$f'(x) = \frac{-2x(x^2-4) + (2x)(x^2)(x^2-1)}{(x^2-1)^2}$$

$$f'(x) = \frac{2x(-x^2+4+x^4-x^2)}{(x^2-1)^2}$$

$$f'(x) = \frac{2x(x^4-2x^2+4)}{(x^2-1)^2} = 0 \rightarrow \boxed{x=0}\checkmark \quad x^4-2x^2+4=0 \quad \Delta<0$$

$$(x^2-1)^2 = 0 \rightarrow x = \pm 1 \text{ غ } \notin D_f$$

x		-2		0		$+2$	
y'	$-$		$+$		$-$		$+$
y	$\searrow$		$\nearrow$		$\searrow$		$\nearrow$

⇐ تعداد اکسترمم نسبی : ۳

$A(0,0) \quad B(1,1) \qquad ab = ?$ -۴

$f(x)$: $y = ax^3 + bx^2 + cx + d$

$A \in f(x) \Rightarrow d = 0$

$B \in f(x) \Rightarrow a + b + \cancel{c} = 1 \Rightarrow a + b = 1 \ *$

$f'(x) = 3ax^2 + 2bx + \cancel{c} = 0$

$\rightarrow c = 0$

$\rightarrow 3a + 2b = 0 \ *$

$\Rightarrow a = -2,\ b = 3 \quad ab = -6$

۵ـ

$x = \pm\sqrt{3}$

$f(x) = x|3 - x^2| \qquad x \in [-1/5 , \sqrt{3}]$ min مطلق؟

-۱۴۰۲-

$f(x) = 3x - x^3$

$f'(x) = 3 - 3x^2 = 3(1 - x^2) = 0 \rightarrow x = \pm 1$

$f(\sqrt{3}) = 0$

$f(-\sqrt{3}) = 0$

$f(1) = 2$

$f(-1) = -2$ min مطلق ✓

$\Rightarrow$ min مطلق $= -2$

۶ـ

$A(-1 , 1)$ $\qquad \frac{b}{a} = ?$

$f(x) = y = x^2|x| + 3ax^2 + b$

$A \in f(x) \Rightarrow 1 + 3a + b = 1 \Rightarrow 3a + b = 0$

$f(x) = x^3 + 3ax^2 + b \rightarrow f'(x) = 3x^2 + 6ax$

$f'(-1) = 0 \rightarrow 3 - 6a = 0 \rightarrow \boxed{a = \frac{1}{2}}$

$\boxed{b = -\frac{3}{2}}$

$\Rightarrow \frac{b}{a} = -3$

۷ـ

$y = \frac{(ax + 3)}{(a+1)x + (a-1)}$

مجانب قائم: $(a+1)x = 1 - a \Rightarrow x = \frac{1-a}{a+1}$

مجانب افقی: $y = \frac{a}{a+1}$

محل تلاقی مجانب‌ها $\left(\frac{1-a}{a+1} , \frac{a}{a+1}\right)$

$y = \frac{3}{2}x^2 + x + \frac{5}{6}$

min

min $\left| \begin{array}{l} \frac{-1}{3} \\ \frac{3}{2}\left(\frac{1}{9}\right) - \frac{1}{3} + \frac{5}{6} = \frac{1}{6} - \frac{2}{6} + \frac{5}{6} = \frac{2}{6} \end{array} \right.$

$\Rightarrow \frac{1-a}{a+1} = \frac{-1}{3} \Rightarrow 3 - 3a = -a - 1 \rightarrow \boxed{a = 2}$

$-2a = -4$

$\frac{2x + 3}{3x + 1} = 0 \rightarrow \boxed{x = \frac{-3}{2}}$

محل تلاقی تابع هموگرافیک با محور xها؟

۸-

$A\left(-\frac{1}{2}, 3\right) \qquad f(x) = y = \dfrac{bx^2+7}{4x^2+ax+1} \qquad \dfrac{b}{a} = ?$

$x = -\frac{1}{2}$ ریشه مخرج $\qquad y = 3$

$1 - \frac{a}{2} + 1 = 0$

$2 = \frac{a}{2} \Rightarrow \boxed{a = 4}$

$* \lim_{x \to \infty} f(x) = \frac{b}{4} = 3 \Rightarrow \boxed{b = 12}$

$\dfrac{b}{a} = \dfrac{12}{4} = 3$

۹-

$f(x) = \dfrac{x^4}{x^3 - 8}$

$f'(x) = \dfrac{4x^3(x^3-8) - \overbrace{(3x^2)(x^4)}^{3x^6}}{(x^3-8)^2}$

$f'(x) = \dfrac{x^3(4x^3 - 32 - 3x^3)}{(x^3-8)^2} = \dfrac{x^3(x^3-32)}{(x^3-8)^2}$

$x^3 \to 0 \qquad x^3 - 32 \to x = \sqrt[3]{32} \qquad (x^3-8)^2 \to x = 2$

	0		$\overset{*}{2}$		$\sqrt[3]{32}$	
y' +	0	−		−	0	+
y ↗		↘		↘		↗

تابع نزولی اکید: $(0, 2) \cup (2, \sqrt[3]{32})$

طول بازه‌ها: $\underbrace{\;}_{2} \qquad \sqrt[3]{32} - 2 = 2\sqrt[3]{4} - 2$

۱۰-

$f(x) = \dfrac{x^4 - 3}{x^2 - 3} \qquad x \in (-2, 2) \qquad D_f: \mathbb{R} - \{\pm\sqrt{3}\}$

$f'(x) = \dfrac{(4x^3)(x^2-3) - (2x)(x^4-3)}{(x^2-3)^2}$

$f'(x) = \dfrac{4x^5 - 12x^3 - 2x^5 + 6x}{(x^2-3)^2} = \dfrac{2x^5 - 12x^3 + 6x}{(x^2-3)^2}$

$f'(x) = \dfrac{2x(x^4 - 6x^2 + 3)}{(x^2-3)^2} = 0 \begin{cases} x = 0 \\ x^4 - 6x^2 + 3 = 0 \to x = \pm\sqrt{3 \pm \sqrt{6}} \end{cases}$

-2	$\overset{*}{-\sqrt{3}}$	$-\sqrt{3-\sqrt{6}}$	0	$\sqrt{3-\sqrt{6}}$	$\sqrt{3}$	2
−	−	+	−	+	+	
↘ ✓	↘ ✓	↗	↘ ✓	↗	↗	

* در سه بازه نزولی اکید