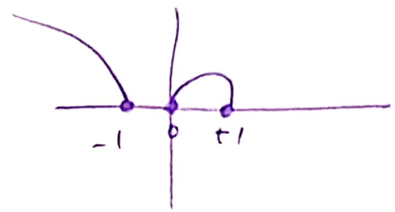
گلشن نیکویی — دوازدهم ریاضی A — تکلیف ۲۶

۱ - رسم می‌کنیم

$\sqrt{x(1+x)}$ ← $x \leq -1$; 0 ; $\sqrt{x(1-|x|)}$ → $\sqrt{x(1-x)}$ → $0 \leq x \leq 1$



$m = 1$

$n = 1$

$k = 4$

$m + n + k = 6$

۲ -

$f(x) = \sqrt{x} + \sqrt{a-2x} \longrightarrow D_f = x \geq 0$ و $-2x + a \geq 0$, $x \leq \frac{a}{2} \to D_f = [0, \frac{a}{2}]$

$f'(x) = \frac{1}{2\sqrt{x}} - \frac{\cancel{2}}{\cancel{2}\sqrt{a-2x}} = 0 \longrightarrow 2\sqrt{x} = \sqrt{a-2x}$

$4x = a - 2x \to x = \frac{a}{6}$

$f(0) = \sqrt{a}$

$f(\frac{a}{6}) = \sqrt{\frac{a}{6}} + \sqrt{\frac{\cancel{4a}}{6}}_{\,2\sqrt{\frac{a}{6}}} = 3\sqrt{\frac{a}{6}}$

$Min \times Max = \sqrt{\frac{a}{2}} \times 3\sqrt{\frac{a}{6}} = \sqrt{12}$

$f(\frac{a}{2}) = \sqrt{\frac{a}{2}}$

$\sqrt{\frac{a}{2} \times \cancel{9}^{3} \times \frac{a}{\cancel{6}_{2}}} = \sqrt{12}$

$\frac{3a^2}{4} = 12 \to 3a^2 = 48$

$4x = a - 2x \to x = \frac{a}{6}$

$f(0) = \sqrt{a}$

$f(\frac{a}{6}) = \sqrt{\frac{a}{6}} + \cancel{\sqrt{\frac{4a}{6}}}$ $2\sqrt{\frac{a}{6}}$ $= 3\sqrt{\frac{a}{6}}$

$f(\frac{a}{2}) = \sqrt{\frac{a}{2}}$

$Min \times Max = \sqrt{\frac{a}{2}} \times 3\sqrt{\frac{a}{6}} = \sqrt{12}$

$\sqrt{\frac{a}{2} \times 9 \times \frac{a}{6}} = \sqrt{12}$

$\frac{3a^2}{4} = 12 \to 3a^2 = 48$

$\frac{9a^2}{12} = 12$

$9a^2 = 144 \to a^2 = \frac{144}{9} = \frac{48}{3} = 16$

$a = 4$

$[a] = [4] = 4$

۳-

یک نقطه (مشترک) $(0,0)$

$x=0$ $\quad -2 \quad 2$

$\frac{x^2}{x^2-1}(x^2-4)$ + (± 2) و نقاط تقاطع

مماس بر محور ها در $x=0$

۳ نقطه

$x = 2$, $x = -2$, $x = 0$

$f(x) = \frac{x^4 - 4x^2}{x^2-1} \Rightarrow f'(x) = \frac{(4x^3 - 8x)(x^2-1) - 2x(x^4-4x^2)}{(x^2-1)^2}$

$4x^5 - 4x^3 - \cancel{8x^3} + 8x - 2x^5 + \cancel{8x^3}$

$= \frac{2x^5 - 4x^3 + 8x}{(x^2-1)^2} = \frac{2x(x^4 - 2x^2 + 4)}{(x^2-1)^2}$

$x^4 - 2x^2 + 4 = 0$ جواب ندارد

$f(x) = \frac{x^2(4 - x^2)}{x^2-1}$ $\quad -x^4 + 4x^2$

$-4x^5 + 4x^3 + \cancel{8x^3} - 8x + 2x^5 - \cancel{8x^3}$

$f'(x) = \frac{(-4x^3 + 8x)(x^2-1) - 2x(-x^4 + 4x^2)}{(x^2-1)^2}$

$-2x^5 + 4x^3 - 8x = 0$

$-2x(x^4 - 2x^2 + 4) = 0 \to \Delta < 0$

$\to x = 0$

۴-

$y = ax^3 + bx^2 + cx + d \to y' = 3ax^2 + 2bx + c$

مشتق = ۰

خود نقطه در تابع صدق کند

$(0,0) \to f'(x) = 0 \to c = 0$ و $d = 0$

$(1,1) \to f'(x) = 0 \to 3a + 2b = 0$

$\hookrightarrow a + b + \cancel{d}^{\,0} = 1 \to a + b = 1$

$2a + 2b = 2$

$3a + 2b = 0$ $\to a = -2$, $b = 3 \to ab = -6$

۵- ضابطه بنویسیم

$f(x) = x|3 - x^2|$

$-\sqrt{3}$ $\quad$ $+\sqrt{3}$

$x(x^2 - 3)$

$f(x) = x^3 - 3x$

$f'(x) = 3x^2 - 3 = 0 \to x = 1$ غیرقابل قبول

$\hookrightarrow x = -1$ قبول

بسته

بعد از $\sqrt{3}$

$x(3 - x^2)$

$f(x) = 3x - x^3$

$f'(x) = 3 - 3x^2$

$x = 1$ ✓

$x = -1$ ✓

$x = 1 \to y = 2$

$x = -1 \to y = -2$

$x = \sqrt{3} \to y = 0$

$x = -\frac{3}{2} \to y = -\frac{3}{2}\left(3 - \frac{9}{4}\right) = -\frac{9}{8}$ غیرقابل قبول

$-2 =$ مطلق Min

$x(x^2-3)$

$f(x)=x^3-3x$

$f'(x)=3x^2-3=0 \to x=1$

$\to x=-1$ عرض نقاط قبول

$-2=$ مطلق Min

نسبت داریم

$x(3-x^2)$

$f(x)=3x-x^3$

$f'(x)=3-3x^2$

$x=1$ ✓

$x=-1$ ✓

$x=1 \to y=2$

$x=-1 \to y=-2$

$x=\sqrt{3} \to y=0$

غرقابل قبول

$x=-\frac{3}{2} \to y=-1/5\left(\frac{3}{\frac{3}{2}}-\frac{9}{4}\right)=-\frac{9}{8}$

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6 -

$f(x)=x^2|x|+3ax^2+b$

$\to x=-1 \to f(x)=-x^3+3ax^2+b$

$\to +1+3a+b=1 \to 3a+b=0$

$\to f'(x)=0 \to -3x^2+6ax=0 \to -3-6a=0$

$-6a=3$

$a=-0/5$

$b=1/5 \to \frac{b}{a}=\frac{+1/5}{-0/5}=-3$

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7 -

$y=\frac{3}{2}x^2+x+\frac{5}{6} \longrightarrow x_s=-\frac{b}{2a}=\frac{-1}{3}$ $\quad \left(-\frac{2}{6},\frac{4}{6}\right)$

$\to y_s=\frac{3}{2}\left(\frac{1}{9}\right)-\frac{1}{3}+\frac{5}{6}=\frac{1}{6}-\frac{1}{3}+\frac{5}{6}=\frac{-1+5}{6}=\frac{4}{6}$

$\frac{a}{a+1}=\frac{2}{3}$

$a=2$

$y=\frac{2x+3}{3x+1} \Rightarrow 2x+3=0$

$x=-\frac{3}{2} \to \left(-\frac{3}{2},0\right)$

۸-

$$y = \frac{bx^2 + 7}{4x^2 + ax + 1} \Rightarrow f(-\tfrac{1}{2}) = \cancel{4}(\tfrac{1}{\cancel{4}}) + a(-\tfrac{1}{2}) + 1 = 0$$

$y=3$ مجانب افقی

$x=-\frac{1}{2}$ مجانب قائم (ریشه مخرج)

$$2 - \frac{a}{2} = 0$$

$a=4$

$$\frac{b}{4} = 3 \rightarrow b = 12$$

$$\frac{b}{a} = 3$$

۹-

$$f(x) = \frac{x^4}{x^3-8} \rightarrow f'(x) = \frac{4x^3(x^3-8) - 3x^2(x^4)}{(x^3-8)^2} = \frac{x^6 - 32x^3}{(x^3-8)^2}$$

$$x^3(x^3-32) = 0$$

$$x = 0 \rightarrow x = \sqrt[3]{32}$$

	0		2		$\sqrt[3]{32}$	
$+$		$-$		$-$		$+$

طول بازه $= 2$ $\quad \sqrt[3]{32} - 2 = 2(\sqrt[3]{4} - 1)$

۱۰-

$$f'(x) = \frac{4x^3(x^2-3) - 2x(x^4-3)}{\ldots} = 4x^5 - 12x^3 - 2x^5 + 6x = 2x^5$$

$$2x^5 - 12x^3 + 6x$$

۹-

$$f(x) = \frac{x^4}{x^3-1} \rightarrow f'(x) = \frac{4x^3(x^3-1) - 3x^2(x^4)}{(x^3-1)^2} = \frac{x^6 - 4x^3}{(x^3-1)^2}$$

$$x^3(x^3 - 4) = 0$$

$$x = 0 \longrightarrow x = \sqrt[3]{4}$$

$$\begin{array}{ccccccc} & 0 & & \cancel{1} & & \sqrt[3]{4} & \\ + & & - & & - & & + \\ \nearrow & & \searrow & & \searrow & & \searrow \end{array}$$

طول بازه $= 2$ ، $2\sqrt[3]{4} - 2 = 2(\sqrt[3]{4} - 1)$

۱۰-

$$f(x) = \frac{x^4-3}{x^2-3} \rightarrow f'(x) = \frac{4x^3(x^2-3) - 2x(x^4-3)}{(x^2-3)^2} = 4x^5 - 12x^3 - 2x^5 + 6x = 2x^5$$

$$2x^5 - 12x^3 + 6x$$

$$2x(x^4 - 6x^2 + 3)$$

$$\Delta = 36 - 12 = 24$$

$$\frac{+6 + 2\sqrt{6}}{2} \qquad \frac{6 - 2\sqrt{6}}{2}$$

$$3 + \sqrt{6} \qquad 3 - \sqrt{6}$$

$$x = \pm\sqrt{5/4} \qquad x^2 = 0/\ldots$$

$$x = \pm\sqrt{0/\ldots}$$

$$\begin{array}{ccccccccccccccc} & -\sqrt{5/..} & & -\cancel{\sqrt{3}} & & -\sqrt{0/..} & & 0 & & \sqrt{0/..} & & \cancel{\sqrt{3}} & & \sqrt{5/..} & \\ - & & + & & + & & - & & + & & - & & - & & + \end{array}$$

۴ بازه صعودی ۳ بازه در دامنه ✓

۱ بازه در دامنه نیست