(۱)

$y=\sqrt{x(1-|x|)}$

$x\geq 0 \rightarrow \sqrt{x(1-x)}=\sqrt{x-x^2} \rightarrow x-x^2\geq 0$; $\begin{array}{c|ccc} & 0 & & 1 \\ \hline & + & + & - \end{array}$

$x\leq 0 \rightarrow \sqrt{x(1+x)}=\sqrt{x+x^2} \rightarrow x+x^2\geq 0$; $\begin{array}{c|ccc} & -1 & & 0 \\ \hline & + & - & + \end{array}$

$\rightarrow D_f=(-\infty,-1]\cup[0,1]$

$x\geq 0 \rightarrow f(x)=\sqrt{x-x^2} \rightarrow f'(x)=\frac{1-2x}{2\sqrt{x-x^2}}=0 \rightarrow x=\frac{1}{2}$ ✓

$x\leq 0 \rightarrow f(x)=\sqrt{x+x^2} \rightarrow f'(x)=\frac{1+2x}{2\sqrt{x+x^2}}=0 \rightarrow x=-\frac{1}{2}$ ✗

x	$\frac{1}{2}$	-1	0	1
y	$\frac{1}{2}$	0	0	0

$\rightarrow \min=0$ ، $\max=\frac{1}{2}$

نقاط بحرانی $\rightarrow \frac{1}{2}$ ، -1 ، 1 ، 0 $\rightarrow$ (۴)

$\rightarrow m+n+k=0+\frac{1}{2}+4=\boxed{4/5}$

(۲)

$f(x)=\sqrt{x}+\sqrt{a-2x}$ $\quad D_f=\begin{cases} x\geq 0 \\ a-2x\geq 0 \rightarrow 2x\leq a \rightarrow x\leq \frac{a}{2} \end{cases} \rightarrow D_f=[0,\frac{a}{2}]$

$\rightarrow f'(x)=\frac{1}{2\sqrt{x}}+\frac{-2}{2\sqrt{a-2x}}=0 \rightarrow \frac{1}{2\sqrt{x}}=\frac{2}{2\sqrt{a-2x}} \rightarrow \sqrt{a-2x}=2\sqrt{x} \rightarrow a-2x=4x \rightarrow 6x=a \rightarrow \boxed{x=\frac{a}{6}}$

$f(0)=\sqrt{a}$

$f(\frac{a}{6})=\sqrt{\frac{a}{6}}=\min$

$f(\frac{a}{6})=\sqrt{\frac{a}{6}}+\sqrt{a-\frac{a}{3}}=\sqrt{\frac{a}{6}}+\sqrt{\frac{2a}{3}}=\sqrt{\frac{a}{6}}(1+2)=\frac{3\sqrt{a}}{\sqrt{6}}=\frac{\sqrt{6a}}{2}=\frac{\sqrt{6}}{2}\sqrt{a}=\max$

$\begin{cases} \frac{\sqrt{6}}{2}\sqrt{a}\times\sqrt{\frac{a}{6}}=\sqrt{12} \\ \rightarrow \frac{\sqrt{3}a}{2}=\sqrt{12} \\ \rightarrow \boxed{a=4} \end{cases}$

$\rightarrow [a]=[4]=\boxed{4}$

(۳)

(۴)

$f(0)=0 \rightarrow 0+0+0+d=0 \longrightarrow \boxed{d=0}$

$f(1)=1 \rightarrow a+b+c+0=1 \longrightarrow a+b+c=1$

$f'(x)=3ax^2+2bx+c \longrightarrow f'(0)=0 \rightarrow \boxed{c=0} \quad f'(1)=0 \rightarrow 3a+2b=0$

$\rightarrow a+b=1 \;,\; 3a+2b=0 \longrightarrow \begin{array}{r} -2a-2b=-2 \\ 3a+2b=0 \\ \hline \boxed{a=-2} \end{array} \;,\; \boxed{b=3} \longrightarrow \boxed{ab=-6}$

(۵) x در بازه $[-1, 0, \sqrt{3}]$ مقدار مشتق صفر ① است

$\rightarrow f(x)=x(3-x^2)=3x-x^3$

$f(-\frac{3}{2})=\frac{-9}{4}-\frac{-27}{8}=\frac{-18}{8}+\frac{27}{8}=\boxed{\frac{9}{8}}$

$f(\sqrt{3})=0$

$f'(x)=3-3x^2=0 \rightarrow 3x^2=3 \rightarrow x=\pm 1$; $x=1 \rightarrow f(1)=2$; $x=-1 \rightarrow f(-1)=-2$ $\rightarrow \boxed{\min_{\text{مطلق}}=-2}$

(۶)

$f(x)=x^2|x|+3ax^2+b \rightarrow f(-1)=1+3a+b=1 \rightarrow 3a+b=0$

$f'(x)=-3x^2+6ax \longrightarrow f'(-1)=-3-6a=0 \rightarrow \boxed{a=-\frac{1}{2}}$; $\boxed{b=\frac{3}{2}}$

$\rightarrow \frac{b}{a}=\frac{\frac{3}{2}}{-\frac{1}{2}}=\boxed{-3}$

(۷)

(۸)

$y=\frac{bx^2+7}{4x^2+ax+1}$

مجانب افقی $=\frac{b}{4} \rightarrow \frac{b}{4}=3 \rightarrow \boxed{b=12}$

مجانب عمودی $=$ ریشه مخرج $=4x^2+ax+1 \xrightarrow{x=-\frac{1}{2}} 1-\frac{a}{2}+1=0 \rightarrow \boxed{a=4}$

$\rightarrow \frac{b}{a}=\frac{12}{4}=\boxed{3}$