مهرانا نازنیلی کلاس ۲۴ «A»

①

$f(x) = \sqrt{x(1-|x|)}$ $\quad f(x) = \begin{cases} \sqrt{x-x^2} & x \geq 0 \\ \sqrt{x+x^2} & x \leq 0 \end{cases}$

$f(x) = \begin{cases} \sqrt{x-x^2} & 0 \leq x \leq 1 \\ \sqrt{x+x^2} & x \leq 0 \end{cases}$ $\quad f'(x) = \begin{cases} \dfrac{-2x+1}{2\sqrt{x-x^2}} & 0 < x \leq 1 \\ \dfrac{2x+1}{2\sqrt{x+x^2}} & x \leq 0 \end{cases}$

بحرانی: $x = 0, 1, -1, \frac{1}{2}$ ④

$m + n + k = 5$

max = 1
min = 0

②

$f(x) = \sqrt{x} + \sqrt{a-2x}$ $\quad f'(x) = \frac{1}{2\sqrt{x}} + \frac{-2}{2\sqrt{a-2x}}$ $\quad D_f = [0, \frac{a}{2}]$

$\frac{1}{2\sqrt{x}} = \frac{1}{\sqrt{a-2x}} \Rightarrow \frac{1}{4x} = \frac{1}{a-2x} \rightarrow a-2x = 4x \rightarrow \frac{a}{6} = x$

x	0	$\frac{a}{2}$	$\frac{a}{6}$
$f(x)$	$\sqrt{a}$	$\sqrt{\frac{a}{2}}$	$3\sqrt{\frac{a}{6}}$

$\max \times \min = 3\sqrt{\frac{a}{6}} \times \sqrt{\frac{a}{2}} = \sqrt{12}$ $\quad a = 4$

③

$f(x) = \frac{x^2 |x^2-4|}{x^2-1} \longrightarrow f(x) \begin{cases} \dfrac{-x^2(x^2-4)}{x^2-1} & -2 \leq x \leq 2 \\ \dfrac{x^2(x^2-4)}{x^2-1} & R - [-2, 2] \end{cases}$

۳ عدد

$f'(x) = \begin{cases} \dfrac{2x(x^4-2x^2+4)}{-(x^2-1)^2} & x \leq -2, \; x \geq 2 \\ \dfrac{2x(-x^4+2x^2-4)}{(x^2-1)^2} & -2 \leq x \leq 2 \end{cases}$

	-2	-1	0	1	2	
$-$	$+$	$+$	$-$	$-$	$+$	

④

$y' = 3ax^2 + 2bx + C$ $\quad d = 0$ $\quad y \rightarrow a + b = 1$

$0 \rightarrow C = 0$ $\quad a - \frac{3}{2}a = -\frac{1}{2}a = 1$

$1 \rightarrow 3a + 2b = 0 \rightarrow \frac{-3a}{2} = b \rightarrow \frac{-3(-2)}{2} = 3$ $\quad b = 3$ $\quad a = -2$

$ab = -6$

⑤

$y = x|3-x^2| \longrightarrow 0, +\sqrt{3}, -\sqrt{3}$

$x \geq \sqrt{3}$ $\quad -x(3-x^2)$
$x \leq -\sqrt{3}$

$-\sqrt{3} \leq x \leq \sqrt{3}$ $\quad x(3-x^2)$

x	0	$\sqrt{3}$	$-\sqrt{3}$	$1/8$	$+1$	-1
$f(x)$	0	0	0	$\frac{-9}{8}$	$+2$	-2

$f'(x) \begin{cases} 3x^2 - 3 \rightarrow x = +1, \; x = -1 \\ -3x^2 + 3 \quad +1, \; -1 \end{cases}$

نقطه min $= (-1, -2)$

(۶)

$x>0 \to x^3+3ax^2+b$ $\quad \frac{b}{a}=\frac{\frac{3}{2}}{-\frac{1}{2}}=(-3)$

$x<0 \to -x^3+3ax^2+b \to -3x^2+6ax \to -3(1)-6a=0$

$1+3a+b=1 \to b=-3(-\frac{1}{2})=\frac{3}{2}=b$ $\quad -3=6a \to -\frac{1}{2}=a$

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(۷)

$y=\frac{3}{2}x^2+x+\frac{5}{6} \to \frac{-1}{2(\frac{3}{2})}=x_{min}=(\frac{-1}{3}) \to \frac{3}{2}(\frac{1}{3})(\frac{1}{3})-\frac{1}{3}+\frac{5}{6}$

$\frac{1}{6}-\frac{1}{3}+\frac{5}{6}=\frac{5}{6}+\frac{2}{6}=(\frac{7}{6}) \to$ مجانب قائم $\to -\frac{1}{3}$

$-\frac{1}{3} \to x=\frac{-a+1}{a+1}=-\frac{1}{3} \to -3a+3=a+1 \to -2a=-4 \to \boxed{a=2}$

$\frac{2x+3}{2x+1}=0 \to \boxed{x=-\frac{3}{2}}$

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(۸)

$y=\frac{bx^2+7}{4x^2+ax+1}$ $\quad A(-\frac{1}{2},3) \to x=-\frac{1}{2} \to$ مجانب قائم / ریشه مخرج $\quad \frac{b}{a}=\frac{12}{4}=(3)$

$4(\frac{1}{4})-\frac{1}{2}a+1=0 \to \frac{1}{2}a=2 \to \boxed{a=4}$ $\quad$ افقی $\to 3$

$\frac{b}{4}=3 \to \boxed{b=12}$

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(۹)

$f'(x)=\frac{(4x^3)(x^3-8)-(3x^2)(x^4)}{(x^3-8)^2} \to =0$

$4x^3(x^3-8)=3x^2(x^4) \to 4x^6-32x^3=3x^6 \to x^3=32 \to \sqrt[3]{2^5}$

$0 \quad 2 \quad \sqrt[3]{2^5}$

$+ \mid + \mid - \mid +$

نزولی $\quad$ نزولی

$(2,\sqrt[3]{2^5}) \to 2\sqrt[3]{4} - 2$ $\quad$ min طول دارد

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(۱۰)

$f'(x)=\frac{(4x^3)(x^2-3)-(2x)(x^4-3)}{(x^2-3)^2}=\frac{4x^5-12x^3-2x^5+6x}{(x^2-3)^2}$

$\pm\sqrt{3}$

$=\frac{2x^5-12x^3+6x}{(x^2-3)^2}$ $\quad 2x(x^4-6x^2+3)$ $\to 0$ ؛ ۴ ریشه ؛ $\pm\sqrt{3}$

$- \mid + \mid + \mid - \mid + \mid - \mid +$

۳ بازه نزولی