سوال ۱۱

$f(x)=\cos^3(2x)+ax^2+b$

$\lim_{x\to0^+}\frac{f(x)}{x}=0 \to \lim_{x\to0^+}\frac{\cos^3(2x)+ax^2+b}{x}=0 \to \lim_{x\to0^+}\frac{1+b}{x}=0 \to \boxed{b=-1}$

$\lim_{x\to0^-}\frac{f'(x)}{x}=2 \to \lim_{x\to0^-}\frac{-6\sin(2x)\cos^2(2x)+2ax}{x}=2 \xrightarrow{\text{هم‌ارزی}} \lim_{x\to0^-}\frac{-6\times2x+2ax}{x}=2$

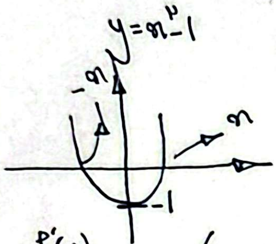
$\to \lim_{x\to0^-}\frac{(2a-12)x}{x}=2 \to 2a-12=2 \to \boxed{a=7}$ $\quad a+b=6$

6

نمره نهایی

$f'(x)=3\cos^2 x\,a-2\sin 2x+2ax$

۱- 0



$\Rightarrow 2(x^2-1)=2x^2-2=\frac{2}{4}-2=\frac12-2=-\frac32$ $\quad f(\frac12)+f(-\frac12)=\frac14-1+\frac14-1=-1.5$

$y'=2x \Rightarrow 2x^2-2x=-1 \Rightarrow x^2=\frac14 \Rightarrow x=\pm\frac12$

۲- 1.5

۳- 0.5

$f'(x)=-\frac{2a}{(2x-1)^2}$ $\quad -\frac{2a}{16}+y=6 \Rightarrow 6+\frac{a}{8}\cdot\frac{25}{...}=y \Rightarrow 18=\frac{a}{8}-\frac{a}{4}=-\frac{25a}{8}$

$-\frac{2a}{4}\,a+y=-12 \quad -12-\frac{a}{4}=y$

$\Rightarrow a=\frac{8\cdot 25}{-25}=-32 \quad \Rightarrow f(0)=-\frac{32}{1-1}=-\frac{32}{9}$

$m=\frac{6-(-12)}{2.5-(-0.5)}=\frac{18}{3}=6$, $6=6(2.5)+b \to b=-9$ $\Big\}\to y=6x-9$ $\quad f(0)=-\frac13$

خط بر منحنی مماس است $\leftarrow \Delta=0$

$\frac{a}{2x-1}=(6x-9) \to 12x^2-18x-6x+9-a=0 \to 12x^2-24x+9-a=0$

$\xrightarrow{\Delta=0} (24)^2-4(12)(9-a)=0 \to 576-432+48a=0 \to 48a=-144 \to a=-3$

۴-

$\frac{1+a}{a+1}=2+b \Rightarrow b=-1$

$\frac{ax+1-a(x+a)}{(ax+1)^2}=2 \Rightarrow \frac{a+1-a-a^2}{(a+1)^2}=2 \Rightarrow 1-a^2=2a^2+2+4a \Rightarrow 3a^2+4a+1=0 \Rightarrow a^2+4a+3=0 \Rightarrow (a+1)(a+3)=0$ $\quad$ 0

مشتق تابع $f'(1)=g'(1) \to \frac{1-a^2}{(a+1)^2}=2 \to \frac{(1-a)(1+a)}{(1+a)^2}=2 \to 2a+2=1-a \to \boxed{a=-\frac13}$

$\Rightarrow a=-1,-3 \Rightarrow a-b=-1+1=0$, $-3+1=-2$ $\quad a-b=\frac23$

مقدار تابع $f(1)=g(1) \to \frac{1-\frac13}{-\frac13+1}=2+b \to \boxed{b=-1}$

۵-

$\frac32\sin x=\sin x+\frac12\cos x$, $\frac12\sin x=\frac12\cos x \Rightarrow x=\frac{\pi}{4}$ $\Rightarrow f'(x)=\cos x-\frac12\sin x \Rightarrow \cos\frac\pi4-\frac12\sin\frac\pi4=\frac{\sqrt2}{2}-\frac{\sqrt2}{4}=\frac{\sqrt2}{4}$ $\quad$ 1

$\Rightarrow \frac32\sin\frac\pi4=\frac32\times\frac{\sqrt2}{2}=\frac{3\sqrt2}{4} \Rightarrow \frac{\sqrt2}{4}\cdot\frac\pi4+y=\frac{3\sqrt2}{4} \Rightarrow y=\frac{3\sqrt2}{4}-\frac{\sqrt2}{4}\cdot\frac\pi4$ $\quad$ (جواب صفحه بعد)

۶-

$f'(x)=6x^2-6x=6x(x-1)$ (0, 1) $\Rightarrow (0,1),(1,-12) \Rightarrow \frac{1+12}{-1}=-13 \Rightarrow -13x+1 \Rightarrow 6x^2-6x=-13 \Rightarrow 6x^2-6x+13=0 \Rightarrow$

$f(1)=2-3-12+1=3-15=-12$ $\quad f'(x)=6x^2-6x-12=-6 \to 6x^2-6x-6=0 \xrightarrow{\Delta>0}$ دو نقطه یافت می‌شود $\quad$ 1.5

۷-

$y=kx^3+(k+1)x^2 \to y'=3kx^2+2(k+1)x \to y''=6kx+2(k+1)=0$ $\quad$ 0

$x=\frac{-k-1}{3k} \xrightarrow[\text{ناحیه دوم}]{\text{نقطه عطف در}} -\frac{k+1}{3k}<0 \to \frac{-\ \ 1\ \ 0}{-\ |\ +\ |\ -} \to \boxed{k<-1,\ k>0\ (I)}$ $\quad (I)\cap(II)\to k>0$

$\to -\frac{k+1}{3k}k+k+1>0 \to -\frac{k+1}{3}+k+1>0 \to \frac{2k+2}{3}>0 \to k+1>0 \to \boxed{k>-1\ (II)}$

طبق مقدار صحیح و منفی نیست

سوال ۱۵

$f(x)=g(x) \rightarrow \sin x+\frac{1}{2}\cos x=\frac{3}{2}\sin x \rightarrow \sin x=\cos x \rightarrow x=\frac{\pi}{4}$

$f(\frac{\pi}{4})=\sin(\frac{\pi}{4})+\frac{1}{2}\cos(\frac{\pi}{4})=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{4}=\frac{3\sqrt{2}}{4}$

$f'(x)=\cos x-\frac{1}{2}\sin x \rightarrow f'(\frac{\pi}{4})=\cos(\frac{\pi}{4})-\frac{1}{2}\sin(\frac{\pi}{4})=\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{4}=\frac{\sqrt{2}}{4}$

$y-f(\frac{\pi}{4})=f'(\frac{\pi}{4})(x-\frac{\pi}{4}) \rightarrow y-\frac{3\sqrt{2}}{4}=\frac{\sqrt{2}}{4}(x-\frac{\pi}{4}) \xrightarrow{y=0} -\frac{3\sqrt{2}}{4}=\frac{\sqrt{2}}{4}(x-\frac{\pi}{4}) \rightarrow x=\frac{\pi}{4}-3$

عطف $x=\frac{-b}{3a} \rightarrow x=\frac{-a}{3} \rightarrow \frac{-a}{3}=-1 \rightarrow \boxed{a=3}$

$x=-1$

$y'=3x^2+2ax+b \Rightarrow 3-2a+b \Rightarrow (3-2a+b)(x+1)+y=-4 \Rightarrow (3-2(-2+b)+b)(x+1)+y=2a-2b$

$x=-1,\ y=-1+a-b-1=-6 \Rightarrow a-b=-2 \Rightarrow a=-2+b$

$3+4-2b+b$

$7-b$

$\frac{a}{b}=\frac{3}{5}$

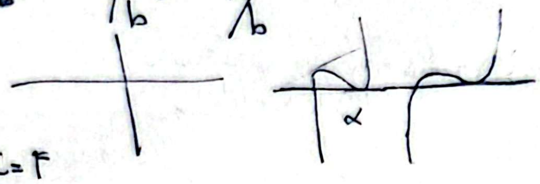
۸

$\Rightarrow -7+b+y=2a-2b \Rightarrow -4+2b-2b=-7b+y \Rightarrow b+y=3 \Rightarrow y=3-b$

$\frac{a}{b}=\frac{-2+b}{b}$

۵

$-4=-1+3-b-1 \rightarrow \boxed{b=5}$



$3x^2+2ax+b=0$

$\Rightarrow 3x^2+(-4+2b)x+b=0 \Rightarrow 3x^2-6x+2bx+b=0$

$x^3+(b-2)x^2+bx-1=-6 \Rightarrow x^3+bx^2-2x^2+bx=-3$

$f(0)=c=4$

$f'(0)=3x^2+2ax+b=0 \Rightarrow b=0$

$3x^2+2ax+b=x(3x+2a)=0 \Rightarrow x=0,\ x=-2a/3$

$\Rightarrow -8a^3/27+4a^3/9+4=0 \Rightarrow -8a^3+12a^3+6a\,27=0$

$\Rightarrow 4a^3+6a\,27=0 \Rightarrow 6(a^3+27)=0 \Rightarrow a=-3 \Rightarrow \min=-2a/3=2$

۲

$f'(x)=4x^3-12x=4x(x^2-3)$, 0, $\pm\sqrt{3}$

$f(\sqrt{3})=f(-\sqrt{3})$

$f''(x)=12x^2-12=12(x^2-1)$, $x=\pm1$

$f(1)=f(-1)$

$f(x)=(x^2-1)(x^2-5)$, ±1, $\pm\sqrt{5}$

x		$-\sqrt{3}$		0		$+\sqrt{3}$	
$f'(x)$	$-$		$+$		$-$		$+$
$f(x)$	↘	-4	↗	5	↘	-4	↗

$9-18+5=-4$

$\Rightarrow$ صفر

۲

-10