مریم توکلی ۲۰ دوازدهم A صبح تکلیف ۲۷

سوال ۱)

$f(x) = \cos^3(2x) + ax^2 + b$ $\quad a+b=?$

$f(0)=0 \xrightarrow{x=0} \cos^3(0) + a(0)^2 + b = 0 \to 1+b=0 \to \underline{b=-1}$

$f'(x) = 3\cos^2(2x) \times (-\sin(2x)) \times 2 + 2ax$

$\to f'(0)=0 \to -3\times 2\sin(2x)\cos^2(2x) + 2ax = 0$ ②

$f''(0)=2 \to -6\left(2\cos(2x)\cos^2(2x) + \underbrace{4\cos(2x)\sin(2x)}\right) + 2a$

$\Rightarrow -12+2a=2 \to \underline{a=7} \quad a+b=7-1=\boxed{6}$

سوال ۲)

$y=x^2-1$ دو نقطه $\to y=c$

$(x_0,c),(-x_0,c)$ $\quad y'=2x$ $\begin{cases} m_{x_0}=2x_0 \\ m_{-x_0}=-2x_0 \end{cases}$

$\to -4x_0^2=-1 \to x_0^2=\frac{1}{4} \to x_0=\pm\frac{1}{2}$ ②

$c=x^2-1 \to x=\frac{1}{2} \to c=-\frac{3}{4}$

$\to x=-\frac{1}{2} \to c=-\frac{3}{4}$

$\Bigg\}$ $\xrightarrow{\text{مجموع}} \boxed{-\frac{3}{2}}$

سوال ۳)

$f(x)=\frac{a}{2x-1}$ $\quad f(5)=?$

شیب خط مماس $=\frac{18}{3}=6$ $\quad f'(x)=\frac{-a(2)}{(2x-1)^2}=\frac{-2a}{(2x-1)^2}$

$(+\frac{5}{2},6) \to y-6=6(x-2/5)$

$\to y-6=6x-15 \to y=6x-9$ ②

$6x-9=\frac{a}{2x-1} \to a=12x^2-24x+9$

$\frac{-2a}{(2x-1)^2}=6 \to -2a=24x^2-24x+6$

$\Bigg\} \to a=1-4x^2$

$\Rightarrow 12x^2-24x+9=1-4x^2 \to 16x^2-24x+8=0$ $\begin{cases} x=1 \to \underline{a=-3} \\ x=\frac{1}{2} \to a=0 \text{ ق ق} \end{cases}$

$f(5)=\frac{-3}{9}=\boxed{-\frac{1}{3}}$

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سوال ۴)

$y = 2x + b \qquad y = \dfrac{x+a}{ax+1}$

$\xrightarrow{x=1} 2 + b = \dfrac{a+1}{\cancel{a+1}} 1 \rightarrow b = -1$

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$y' = 2 \qquad y' = \dfrac{1(ax+1) - (x+a)(a)}{(ax+1)^2} \xrightarrow{x=1} \dfrac{1-a}{a+1} = 2$

$(a+1)^2$

$a - b = -\dfrac{1}{3} + 1 = \boxed{\dfrac{2}{3}} \qquad \rightarrow 2a + 2 = 1 - a \Rightarrow 3a = -1$

$\Rightarrow a = \dfrac{-1}{3}$

سوال ۵)

$f(x) = \sin x + \dfrac{1}{2}\cos x \qquad g(x) = \dfrac{3}{2}\sin x$

$f(x) = g(x) \rightarrow \sin x + \dfrac{1}{2}\cos x = \dfrac{3}{2}\sin x \Rightarrow \cancel{\tfrac{1}{2}}\cos x = \cancel{\tfrac{1}{2}}\sin x$

$\Rightarrow \cos x = \sin x \Rightarrow x = \dfrac{\pi}{4} \rightarrow f(\dfrac{\pi}{4}) = \dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{4} = \dfrac{3\sqrt{2}}{4}$

$f'(x) = \cos x - \dfrac{1}{2}\sin x \rightarrow f'(\dfrac{\pi}{4}) = \dfrac{\sqrt{2}}{2} - \dfrac{\sqrt{2}}{4} = \dfrac{\sqrt{2}}{4}$

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$y - \dfrac{3\sqrt{2}}{4} = \dfrac{\sqrt{2}}{4}(x - \dfrac{\pi}{4}) \rightarrow y - \dfrac{3\sqrt{2}}{4} = \dfrac{\sqrt{2}x}{4} - \dfrac{\sqrt{2}\pi}{16}$

$\Rightarrow y = \dfrac{\sqrt{2}x}{4} + \dfrac{12\sqrt{2} - \sqrt{2}\pi}{16} \xrightarrow{y=0} \dfrac{\cancel{\sqrt{2}}x}{\cancel{4}} = \dfrac{\sqrt{2}\pi - 12\sqrt{2}}{\frac{16}{4}}$

$\Rightarrow 4x = \pi - 12 \rightarrow \boxed{x = \dfrac{\pi - 12}{4}}$

سوال ۶)

$f(x) = 2x^4 - 4x^3 - 12x + 1$

$f'(x) = 4x^2 - 4x - 12 = 0 \rightarrow x^2 - x - 3 = 0 \rightarrow (x-2)(x+1) = 0$

$x = 2 \quad x = -1$

$x = 2 \rightarrow f(x) = 16 - 12 - 24 + 1 = -19$

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$x = -1 \rightarrow f(x) = -2^2 - 2 + 12 + 1 = 8 \qquad m_{AB} = \dfrac{8-(-19)}{-1-2} = -9$

$4x^2 - 4x - 12 = -9 \rightarrow 4x^2 - 4x - 3 = 0 \rightarrow 2x^2 - 2x - 1 = 0$

$x = \dfrac{2 \pm 2\sqrt{3}}{4} \left\langle \begin{array}{l} \dfrac{1+\sqrt{3}}{2} \\ \dfrac{1-\sqrt{3}}{2} \end{array} \right\} \rightarrow \boxed{\text{۲ نقطه}}$

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سوال ۷)

$y = kx^3 + (k+1)x^2$

$y' = 3kx^2 + 2(k+1)x \rightarrow y'' = 6kx + 2(k+1) = 6kx + 2k + 2$

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$6kx + 2k + 2 = 0 \rightarrow x = \frac{-k-1}{3k}$ $\quad \underset{\text{شرط ۲}}{x<0}$ $\quad - \; + \; -$ ①

$f\left(\frac{-k-1}{3k}\right) > 0 \rightarrow k\left(\frac{-k-1}{3k}\right)^3 + (k+1)\left(\frac{-k-1}{3k}\right)^2 > 0$

شرط ۲

$\rightarrow \frac{2(k+1)^3}{27k^2} > 0 \Rightarrow k > -1$ ②

در ① ∩ ② هیچ مقدار منفی برای k نداریم!

($27k^2$: همواره +)

سوال ۸)

$y = x^3 + ax^2 + bx - 1 \qquad \frac{a}{b}$

نقطه $(-1, -4)$ نقطه عطف است

$x = -1 \rightarrow -1 + a - b - 1 = -4 \rightarrow a - b = -2$

عطف تابع هست!

$y' = 3x^2 + 2ax + b \qquad y'' = 6x + 2a$

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$\xrightarrow{x=-1} -6 + 2a = 0 \rightarrow a = 3 \qquad 3 - b = -2 \rightarrow b = 5$

$\frac{a}{b} = \boxed{\frac{3}{5}}$

سوال ۹)

$f(x) = x^3 + ax^2 + bx + c$

$(0,4) \rightarrow c = 4 \qquad f'(x) = 3x^2 + 2ax + b \xrightarrow{f'(0)=0} b = 0$

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$\min = (x_0, 0) \longrightarrow f(x_0) = x_0^3 + ax_0^2 + 4$

$f'(x_0) = 3x_0^2 + 2ax_0 = 0 \rightarrow x_0(3x_0 + 2a) = 0$

غ ق ق $x_0 = 0$ $\qquad x_0 = -\frac{2a}{3}$ ق ق

$\rightarrow -\frac{8a^3}{27} + \frac{4a^3}{9} + 4 = 0$

$\frac{12a^3}{27} \qquad \frac{4a^3}{27} \searrow \frac{4(a^3+27)}{27} = 0 \rightarrow a = -3$

$x_0 = \frac{-2(-3)}{3} = \boxed{2}$

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سوال ۱۰)

$f(x) = x^4 - 6x^2 + 5$

$f'(x) = 4x^3 - 12x \rightarrow 4x(x^2 - 3)$

$4x = 0 \downarrow 0 \qquad x^2 - 3 \downarrow \pm\sqrt{3}$

x		$-\sqrt{3}$		0		$\sqrt{3}$	
y'	$-$		$+$		$-$		$+$
y	$\searrow$	min نسبی	$\nearrow$		$\searrow$	min نسبی	$\nearrow$

$x = -\sqrt{3} \rightarrow y = 9 - 18 + 5 = \underline{-4} \Rightarrow A$

$x = \sqrt{3} \rightarrow y = -4 \Rightarrow B \qquad \rightarrow \underline{m_{AB} = 0}$

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$f''(x) = 12x^2 - 12 \rightarrow 12(x^2 - 1) \rightarrow \pm 1$

$x = +1 \rightarrow y = 0$

$x = -1 \rightarrow y = 0 \rightarrow \underline{m_{CD} = 0}$

زاویه بین دو پاره خط $\boxed{\text{صفر}}$

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