مریم توکلی دوازدهم A تکلیف ۲۷

سوال ۱)

$f(x)=\cos^3(\pi x)+ax^2+b \qquad a+b=5$

$f(0)=0 \xrightarrow{x=0} \cos^3(0)+a(0)^2+b=0 \to 1+b=0 \to b=-1$

$f'(x)=3\cos^2(\pi x)\times(-\sin(\pi x))\times\pi+2ax$

$\to f'(0)=0 \to -3\times\pi\sin(\pi x)\cos^2(\pi x)+2ax=0$

$f''(x)=2 \to -9\left(\pi\cos(\pi x)\cos^2(\pi x)+\pi\cos(\pi x)\sin(\pi x)\right)+2a$

$\Rightarrow -12\pi+2a=2 \to a=6 \qquad a+b=6-1=\boxed{5}$

سوال ۲)

$y=x^2-1 \qquad$ مشتق $\to y=c$

$(x_0,c),(-x_0,c) \qquad y'=2x \begin{cases} m_{x_0}=2x_0 \\ m_{-x_0}=-2x_0 \end{cases}$

$\to -4x_0^2=-1 \to x_0^2=\frac{1}{4} \to x_0=\pm\frac{1}{2}$

$c=x^2-1 \to x=\frac{1}{2} \to c=-\frac{3}{4}$

$x=-\frac{1}{2} \to c=-\frac{3}{4}$ } مجموع $\to \boxed{-\frac{3}{2}}$

سوال ۳)

$f(x)=\frac{a}{2x-1} \qquad f(3)=?$

شیب مماس $=\frac{12}{3}=4 \qquad f'(x)=\frac{-a(2)}{(2x-1)^2}=\frac{-2a}{(2x-1)^2}$

$\left(+\frac{3}{2},4\right) \to y-4=4(x-1/5)$

$\to y-4=4x-10 \to y=4x-6$

$4x-6=\frac{a}{2x-1} \to a=12x^2-24x+6$

$\frac{-2a}{(2x-1)^2}=4 \to -2a=32x^2-32x+8$ } $\to a=1-4x^2$

$\Rightarrow 12x^2-24x+6=1-4x^2 \to 16x^2-24x+5=0 \begin{cases} x=1 \to a=-3 \\ x=\frac{1}{2} \to a=0 \text{ ق ق} \end{cases}$

$f(3)=\frac{-3}{9}=\boxed{-\frac{1}{3}}$

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سوال ۴)

$y = 2x + b \qquad y = \frac{x+a}{ax+1}$

$x=1 \rightarrow 2+b = \frac{a+1}{a+1} \rightarrow b = -1$

$y' = 2 \qquad y' = \frac{(ax+1) - (x+a)(a)}{(ax+1)^2} \xrightarrow{x=1} \frac{1-a}{(a+1)^2} = 2$

$\rightarrow 2a+2 = 1-a \Rightarrow 3a = -1$

$\Rightarrow a = \frac{-1}{3}$

$a - b = -\frac{1}{3} + 1 = \boxed{\frac{2}{3}}$

سوال ۵)

$f(x) = \sin x + \frac{1}{2}\cos x \qquad g(x) = \frac{3}{2}\sin x$

$f(x) = g(x) \rightarrow \sin x + \frac{1}{2}\cos x = \frac{3}{2}\sin x \Rightarrow \frac{1}{2}\cos x = \frac{1}{2}\sin x$

$\Rightarrow \cos x = \sin x \rightsquigarrow x = \frac{\pi}{4} \rightarrow f(\frac{\pi}{4}) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{4} = \frac{3\sqrt{2}}{4}$

$f'(x) = \cos x - \frac{1}{2}\sin x \rightarrow f'(\frac{\pi}{4}) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{4} = \frac{\sqrt{2}}{4}$

$y - \frac{3\sqrt{2}}{4} = \frac{\sqrt{2}}{4}(x - \frac{\pi}{4}) \rightarrow y - \frac{3\sqrt{2}}{4} = \frac{\sqrt{2}x}{4} - \frac{\sqrt{2}\pi}{16}$

$\Rightarrow y = \frac{\sqrt{2}x}{4} + \frac{12\sqrt{2} - \sqrt{2}\pi}{16} \xrightarrow{y=0} \frac{\sqrt{2}x}{4} = \frac{\sqrt{2}\pi - 12\sqrt{2}}{16}$

$\Rightarrow 4x = \pi - 12 \rightarrow \boxed{x = \frac{\pi - 12}{4}}$

سوال ۶)

$f(x) = 2x^4 - 4x^2 - 12x + 1$

$f'(x) = 4x^2 - 4x - 12 = 0 \rightarrow x^2 - x - 3 = 0 \rightarrow (x-2)(x+1) = 0$

$x = 2 \quad x = -1$

$x = 2 \rightarrow f(x) = 16 - 16 - 24 + 1 = -19$

$x = -1 \rightarrow f(x) = -4 - 4 + 12 + 1 = 5 \qquad m_{AB} = \frac{5 - (-19)}{-1 - 2} = \frac{-9}{3}$

$4x^2 - 4x + 12 = -9 \rightarrow 4x^2 - 4x - 3 = 0 \rightarrow 2x^2 - 2x - 1 = 0$

$x = \frac{2 \pm 2\sqrt{3}}{4} \left\{ \begin{matrix} \frac{1+\sqrt{3}}{2} \\ \frac{1-\sqrt{3}}{2} \end{matrix} \right\} \rightarrow \boxed{\text{جواب}}$

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سوال ۷)

$y = kx^3 + (k+1)x^2$

$y' = 3kx^2 + 2(k+1)x \rightarrow y'' = 6kx + 2(k+1) = 6kx + 2k + 2$

$6kx + 2k + 2 = 0 \rightarrow x = \frac{-k-1}{3k}$ $x<0$ ۲ تغییر علامت ①

$f(\frac{-k-1}{3k}) > 0 \rightarrow k(\frac{-k-1}{3k})^3 + (k+1)(\frac{-k-1}{3k})^2 > 0$

تغییر علامت ۲

$\rightarrow \frac{2(k+1)^3}{27k^2} > 0 \rightarrow \underline{k > -1}$ ② در ① ∩ ② هیچ

همواره + مقدار منفی برای k نداریم!

سوال ۸)

$y = x^3 + ax^2 + bx - 1$ $\frac{a}{b}$

نقطه (-۱,-۴) نقطه عطف

$x = -1 \rightarrow -1 + a - b - 1 = -4 \rightarrow a - b = -2$

عطف تابع هست!

$y' = 3x^2 + 2ax + b$ $y'' = 6x + 2a$

$\xrightarrow{x=-1} -6 + 2a = 0 \rightarrow \underline{a = 3}$ $3 - b = -2 \rightarrow \underline{b = 5}$

$\frac{a}{b} = \boxed{\frac{3}{5}}$

سوال ۹)

$f(x) = x^3 + ax^2 + \cancel{bx} + c$ ۴

$(0,4) \rightarrow \underline{c = 4}$ $f'(x) = 3x^2 + 2ax + b \xrightarrow{f'(0)=0} \underline{b = 0}$

min نسبی $= (x_0, 0) \rightarrow f(x_0) = x_0^3 + ax_0^2 + 4$

$\searrow f'(x_0) = 3x_0^2 + 2ax_0 = 0 \rightarrow x_0(3x_0 + 2a) = 0$

غ ق ق $x_0 = 0$ ، ق ق $x_0 = -\frac{2a}{3}$

$\rightarrow -\frac{8a^3}{27} + \frac{4a^3}{9} + 4 = 0$ ، $\frac{12a^3}{27}$ ، $\frac{108}{27}$ $\searrow \frac{4(a^3+27)}{27} = 0 \rightarrow a = -3$

$x_0 = \frac{-2(-3)}{3} = \boxed{2}$

سوال ۱۰

$f(x) = x^4 - 4x^2 + 5$

$f'(x) = 4x^3 - 8x \rightarrow 4x(x^2 - 2)$

$x = 0, \pm\sqrt{2}$

	$-\sqrt{2}$		0		$\sqrt{2}$	
y'	$-$	$+$		$-$		$+$
y	↘	↗		↘		↗
	min				min	

$x = -\sqrt{2} \rightarrow y = 9 - 18 + 5 = -4 \Rightarrow A$

$x = \sqrt{2} \rightarrow y = -4 \Rightarrow B \rightarrow m_{AB} = 0$

$f''(x) = 12x^2 - 12 \rightarrow 12(x^2 - 1)$

$x = \pm 1$

$x = +1 \rightarrow y = 0$

$x = -1 \rightarrow y = 0 \rightarrow m_{CD} = 0$

زاویه بین دو خط صفر