تمرین کنید

①

$f(x) = \cos^3(2x) + ax^2 + b$ $\lim_{x\to 0^+} \frac{f(x)}{x} = 0$ $\lim_{x\to 0^-} \frac{f'(x)}{x} = 2$ $a+b=?$ ۶

$f' = (-2\sin 2x)\times 3 \times \cos^2 2x + 2ax$ $\lim_{x\to 0^+} \frac{f}{x} = 0 \to f(0^+) = 0 \to 1+b=0 \to b=-1$

$f' = -6\sin 2x\cos^2 2x + 2ax$

$f'' = -12\cos 2x\cos 2x + 6\sin 2x\sin 4x + 2a$ $\lim_{x\to 0^+} \frac{f'}{x} = 2 \xrightarrow{hop} \lim_{x\to 0^+} f'' = 2$

$\xrightarrow{x=0^+} -12 + 2a = 2 \to a = 7$

②

$y = x^2 - 1$ تلاقی در دو نقطه

$d: y = d$ → مماس های دو نقطه بر هم عمودند → مجموع عرض ها ?

$x^2 - 1 = d \to x^2 = d+1 \to x = \sqrt{d+1}$, $x = -\sqrt{d+1}$

$y' = 2x$; $y' = 2\sqrt{d+1}$, $y' = -2\sqrt{d+1}$; $m = -\frac{1}{m'}$; $2\sqrt{d+1} = \frac{1}{2\sqrt{d+1}} \to 4(d+1) = 1 \to d+1 = \frac{1}{4} \to d = -\frac{3}{4}$

مجموع عرض ها : $2\left(-\frac{3}{4}\right) = -\frac{3}{2}$

③

$f = \frac{a}{2x-1}$ → خط مماس از → $f(5) = ?$

$\left(\frac{5}{2}, 6\right) , \left(-\frac{1}{2}, -12\right)$

عبور میکند

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-12-6}{-\frac{1}{2} - \frac{5}{2}} = \frac{-18}{-3} = 6 \to g(x) = 6x + b \xrightarrow[y=6]{x=\frac{5}{2}} 6 = 15 + b \to b = -9 \to g(x) = 6x - 9$

$f' = \frac{-2a}{(2x-1)^2} = 6 \to \frac{a}{(2x-1)^2} = -3 \to a = -3(2x-1)^2$

$f = g \to \frac{a}{2x-1} = 6x - 9 \to 3(2x-3)(2x-1) = a$

$3(2x-3)\cancel{(2x-1)} = -3\cancel{(2x-1)}(2x-1)$

$2x - 3 = -2x + 1 \to 4x = 4$

$x = 1$

$\xrightarrow{x=1} -3 = a \to f(x) = \frac{-3}{2x-1} \to f(5) = \frac{-3}{9} = -\frac{1}{3}$

s.a.m

(۴)

$y = 2x + b$ $\qquad$ $y = \dfrac{x+a}{ax+1}$ $\quad$ ← این دو خط در نقطه‌ای به طول ۱ مماس اند ← $a-b=?$

$x=1$ ←

$f = g \rightarrow 2(1) + b = \dfrac{1+a}{a+1} \rightarrow 2 + b = 1 \rightarrow \boxed{b = -1}$

$x=1$ ←

$g' = f' \rightarrow 2 = \dfrac{ax+1-ax-a^2}{(ax+1)^2} \rightarrow 2 = \dfrac{1-a^2}{(ax+1)^2} \xrightarrow{x=1} 2 = \dfrac{1-a^2}{(a+1)^2} \rightarrow \dfrac{1-a}{1+a} = 2$

$\rightarrow 2 + 2a = 1 - a \rightarrow 3a = -1 \rightarrow \boxed{a = \dfrac{-1}{3}}$ $\qquad$ $a - b = \dfrac{-1}{3} + \dfrac{3}{3} = \boxed{\dfrac{2}{3}}$

(۵) $f = \sin + \dfrac{\cos}{2}$, $g = \dfrac{3\sin}{2}$ $\quad [0, \pi]$ ← در بازه خط مماس بر f ... محل برخورد ← این خط مماس ... محور x ها را در چه نقطه‌ای قطع می‌کند

$\sin + \dfrac{\cos}{2} = \dfrac{3}{2}\sin \rightarrow \dfrac{\sin}{2} = \dfrac{\cos}{2} \rightarrow \sin = \cos \rightarrow x = \dfrac{\pi}{4}$

$f' = \cos - \dfrac{\sin}{2} \xrightarrow{x=\frac{\pi}{4}} f' = \dfrac{2\sqrt{2}}{4} - \dfrac{\sqrt{2}}{4} = \dfrac{\sqrt{2}}{4}$

$f\left(\dfrac{\pi}{4}\right) = \dfrac{2\sqrt{2}}{4} + \dfrac{\sqrt{2}}{4} = \dfrac{3\sqrt{2}}{4}$

$\rightarrow y = \dfrac{\sqrt{2}}{4}x + b \xrightarrow[y=\frac{3\sqrt{2}}{4}]{x=\frac{\pi}{4}} \dfrac{12\sqrt{2}}{16} = \dfrac{\sqrt{2}\pi}{16} + b$

$\boxed{b = \dfrac{\sqrt{2}(12-\pi)}{16}}$

$y = 0 \rightarrow \dfrac{\sqrt{2}}{\cancel{4}}x = \dfrac{\sqrt{2}\pi - 12\sqrt{2}}{\cancel{16}4} \rightarrow \boxed{x = \dfrac{\pi - 12}{4}}$

(۶) $f(x) = 2x^3 - 3x^2 - 12x + 1$ ← نقاط اکسترمم A, B ← خط مماس موازی AB در چند نقطه

$f' = 6x^2 - 6x - 12$ $\qquad$ $\begin{cases} x = 2 \rightarrow y = -19 \\ x = -1 \rightarrow y = 14 \end{cases}$ $\qquad$ $m_{AB} = \dfrac{14+19}{-1-2} = -11$

$\rightarrow 6x^2 - 6x - 12 = -11 \rightarrow 6x^2 - 6x - 1 = 0 \rightarrow \Delta > 0 \rightarrow$ ۲ نقطه

(۷) $y = kx^3 + (k+1)x^2$ ← نقطه عطف در ربع دوم ← محدوده k

$y' = 3kx^2 + 2kx + 2x \rightarrow y'' = 6kx + 2k + 2 \rightarrow y'' = 0 \rightarrow x = \dfrac{-2k-2}{6k} \xrightarrow{\text{ربع ۲}} x < 0$

$\dfrac{-2k-2}{6k} < 0$ $\qquad$ $\begin{array}{c|c|c} & 0 & 1 \\ - & + & - \end{array}$ $\qquad$ $k > 1$, $k < 0$

$y = x^2(kx + k + 1) \xrightarrow{x = \frac{-2k-2}{6k}} \left(\dfrac{-2k-2}{6k}\right)^2\left(\dfrac{-2k-2}{6} + k + 1\right) > 0 \xrightarrow[y>0]{\text{ربع ۲}} \dfrac{-k-1+3k+3}{3} >$

$\dfrac{2k+2}{3} > 0 \rightarrow 2k + 2 > 0 \rightarrow k > -1 \xrightarrow{\text{s.a.m}} -1 < k < 0$, $k > 1$ → صحیح

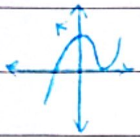
(8)

$(-1,-4)$

$y = x^3 + ax^2 + bx - 1$ → [illegible] → $\frac{a}{b} = ?$

$f(-1) = -4 \rightarrow -1 + a - b - 1 = -4 \rightarrow a - b = -2$

(9) $f(x) = x^3 + ax^2 + bx + c$



→ طول max نسبی ?

$f(0) = 4 \rightarrow c = 4$

$f'(0) = 0 \rightarrow 3x^2 + 2ax + b = 0 \rightarrow b = 0$ $\Big\}$ → $f(x) = x^3 + ax^2 + 4$

$f'(x) = 3x^2 + 2ax = x(3x + 2a)$

s.a.m