اما ارشری ۱۴

$f(x) = \cos^4(2x) + ax^2 + b \ , \ f'(x) = -4 \times 2 \sin^2(2x) + 2ax$

①

$\lim_{x\to 0^+} \frac{f(x)}{x} = \lim_{x\to 0^+} \frac{\cos^4 2x + ax^2 + b}{x} \longrightarrow \lim_{x\to 0^+} \frac{1x^2 + ax^2 + b}{x} = 0 \to b = 0$

$f(x) = \cos^4(2x) + ax^2 + b$

$\lim_{x\to 0^-} \frac{f'(x)}{x} = \lim_{x\to 0^-} \frac{-4\sin^2(2x) + 2ax}{x} \longrightarrow \lim_{x\to 0^-} \frac{-4\times 4x^2 + 2ax}{x} \to \lim_{x\to 0^-} \frac{-24x + \dots}{\dots}$

$\lim_{x\to 0^+} \frac{f(x)}{x} = 0 \longrightarrow \lim_{x\to 0^+} \frac{\cos^4(2x) + ax^2 + b}{x} = 0 \longrightarrow \lim_{x\to 0^+} \frac{1+b}{x} = 0 \longrightarrow \boxed{b=-1}$

$a+b=6$

$\longrightarrow \lim_{x\to 0} -24x + 2a = 2 \to \boxed{a=1}$

$a + b \Rightarrow 1 + 0 = 1$ ①

$\lim_{x\to 0^-} \frac{f'(x)}{x} = 2 \to \lim_{x\to 0^-} \frac{-8\sin(2x)\cos^3(2x) + 2ax}{x} = 2 \xrightarrow{\sin \text{ هم‌ارزی}} \lim_{x\to 0^-} \frac{-8\times 2x + 2ax}{x} = 2$

$\longrightarrow \lim_{x\to 0^-} \frac{(2a-16)x}{x} = 2 \longrightarrow 2a - 16 = 2 \longrightarrow \boxed{a=9}$

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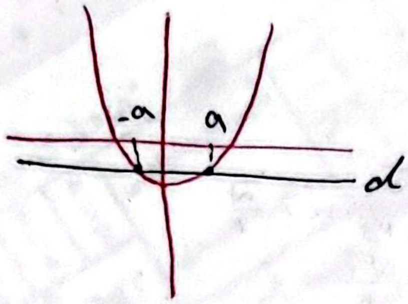
②



$f'(x) = x^2 - 1$

$\hookrightarrow f'(x) = 2x$

$f'(a) \cdot f'(-a) = -1$

$\hookrightarrow 2a \times (-2a) = -1 \longrightarrow a^2 = 1/4 \longrightarrow a = \pm\frac{1}{2}$

$\xrightarrow{a>0} a = \frac{1}{2}$

$f(x) = x^2 - 1 \xrightarrow{a=1/2} \frac{1}{4} - 1 = \frac{-3}{4}$

$\xrightarrow{a=-1/2} \frac{1}{4} - 1 = \frac{-3}{4}$

$\longrightarrow -\frac{3}{4} \times 2 = \boxed{-1/5}$ جواب

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③

$A \left| \begin{matrix} 2/5 \\ 6 \end{matrix} \right. \quad B \left| \begin{matrix} -0/5 \\ -12 \end{matrix} \right. \quad m_{AB} = \frac{6-(-12)}{2/5-(-0/5)} = \frac{18}{3} = 6 \quad y = 6x - 9$

$f(x) = \frac{a}{2x-1} \longrightarrow f'(x) = \frac{-2a}{(2x-1)^2} \longrightarrow f'(x) = \frac{-2a}{(4x^2+1-4x)} = 6$

$f(x) = \frac{a}{2x-1} = 6x - 9$

$\longrightarrow \begin{cases} a = -3(2x-1)^2 \\ a = (2a-1)(6x-9) \end{cases} \Rightarrow -3(2x-1) = 6x - 9 \longrightarrow -6x + 3 = 6x - 9$

$12x = 12 \longrightarrow \boxed{x=1}$

$a = -3 \longrightarrow f(0) = \frac{-3}{9} = -\frac{1}{3}$

(۴)

$$f(x)=\frac{x+a}{ax+1}\longrightarrow f'(x)=\frac{1(ax+1)-(a)(x+a)}{(ax+1)^2}$$

$$\frac{ax+1-ax-a^2}{(ax+1)^2}=\frac{1-a^2}{(ax+1)^2}\longrightarrow y'(1)=\frac{1-a^2}{(a+1)^2}=2\longrightarrow 2(a^2+2a+1)=1-a^2$$

$$2a^2+4a+2-1+a^2=0\longrightarrow 3a^2+4a+1\longrightarrow (a+1)(3a+1)=0\begin{cases}a=-1 \times\\ a=-1/3 \checkmark\end{cases}$$

(۲)

$$y=\frac{x-\frac{1}{3}}{1/3x+1}\longrightarrow\frac{3x-1}{-x+3}\xrightarrow{x=1}\frac{3-1}{-1+3}=1$$

$$1=2+b\longrightarrow b=-1\longrightarrow a-b=-\frac{1}{3}+1=\frac{2}{3}$$ ✓

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(۵)

$$f(x)=g(x)\longrightarrow\frac{3}{2}\sin x=\sin x+\frac{1}{2}\cos x$$

$$1/2\sin x=\frac{1}{2}\cos x\longrightarrow\sin x=\cos x\xrightarrow[\,[0,\frac{\pi}{2}]\,]{x\ \text{در بازه}}x=\frac{\pi}{4}$$

(۲)

$$f'(x)=\cos x-1/2\sin x\longrightarrow f'(\frac{\pi}{4})=\frac{2\sqrt{2}}{2\cdot 4}\cdot\frac{-\sqrt{2}}{4}=\frac{\sqrt{2}}{4}$$

$$f(\frac{\pi}{4})=\frac{2\sqrt{2}}{2\cdot 4}+\frac{\sqrt{2}}{4}=\frac{3\sqrt{2}}{4}\longrightarrow y=\frac{\sqrt{2}}{4}x+b\xrightarrow{x=\frac{\pi}{4}}\frac{\sqrt{2}}{4}\times\frac{\pi}{4}+b=\frac{3\sqrt{2}}{4}$$

$$\longrightarrow b=-\frac{\pi\sqrt{2}}{16}+\frac{3\sqrt{2}}{4}\xrightarrow{y=0}\frac{\sqrt{2}}{4}x+\frac{3\sqrt{2}}{4}-\frac{\pi\sqrt{2}}{16}=0\longrightarrow x=\frac{\pi}{4}-3$$

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(۶)

$$f(x)=2x^3-3x^2-12x+1\longrightarrow f'(x)=6x^2-6x-12$$

(۱٫۵)

$$\xrightarrow{\div 6}x^2-x-2\longrightarrow(x-2)(x+1)$$

$x=2,\ -1$

	-1		2	
	+	−	+	
	↗ 8	↘	-19 ↗	

$A\begin{vmatrix}2\\-19\end{vmatrix}\quad B\begin{vmatrix}-1\\8\end{vmatrix}\quad m_{AB}=\frac{8-(-19)}{-1-2}=\frac{27}{-3}=\boxed{-9}$

$$y=-9x+b\longrightarrow 8=9+b\longrightarrow b=-1$$

$$-9x-1=2x^3-3x^2-12x+1\longrightarrow 2x^3-3x^2-3x+2$$

$$\xrightarrow{x=-1}-2+3-3+2=0$$

	2	-3	-3	2
-1		-2	5	-2
	2	-5	2	

$$(x+1)(\underbrace{2x^2-5x+2}_{x^2+5x+4 \,\to\, (x+\frac{5}{2})(2x+1)})\longrightarrow(x+1)(x+2)(2x+1)\begin{cases}x=-1\\x=-2\\x=-1/2\end{cases}$$

۳ نقطه

$$f'(x)=6x^2-6x-12=-9\longrightarrow 6x^2-6x-3=0\xrightarrow{\Delta>0}$$ دو نقطه یافت می‌شود

۱۷ (۱)

$y' = 3kx^2 + 2(k+1)x \xrightarrow{\text{مشتق}} y'' = 6kx + 2(k+1) = 0$

$x = \dfrac{-k-1}{3k}$

حالت I: $x < 0 \Rightarrow \dfrac{-k-1}{3k} < 0 \xrightarrow{\text{تعیین علامت}}$ (جدول علامت: -1 ، 0) $\longrightarrow$ بازه‌ی k $\rightarrow (-\infty, -1) \cup (0, +\infty)$ ✓

حالت II: $y = k \times \left(\dfrac{-k-1}{3k}\right)^3 + (k+1)\left(\dfrac{-k-1}{3k}\right)^2 > 0 \longrightarrow \dfrac{-(k+1)^3}{9k^2} + \dfrac{-(k+1)^3}{27k}$

$\longrightarrow \dfrac{-k(k+1)^3}{27k^2} > 0 \xrightarrow{\text{تعیین علامت}}$ (جدول علامت: -1 ، 0 مضاعف) $\longrightarrow$ بازه‌ی k $\rightarrow (-\infty, -1)$

اشتراک $I \cap II \longrightarrow k \in (-\infty, -1) \longrightarrow$ نادرست!

$\longrightarrow -\dfrac{k+1}{3k}k + k + 1 > 0 \rightarrow -\dfrac{k+1}{3} + k + 1 > 0 \rightarrow \dfrac{2k+2}{3} > 0 \rightarrow k + 1 > 0 \rightarrow k > -1$ (II)

(I) $\cap$ (II) $\rightarrow k > 0 \rightarrow$ هیچ مقدار صحیح و منفی نیست

(8) ①

$f'(x) \to 3x^2+2ax+b \xrightarrow{x=-1} 3-2a+b=0 \to b-2a=-3$

$f(x) = x^3+ax^2+bx-1 \xrightarrow{(-1,-4)} -1+a-b-1=-4 \to a-b=-4+2$

$a-b=-2$

$\begin{cases} b-2a=-3 \to b-10=-3 \to b=7 \\ a-b=-2 \end{cases}$

$-a=-5 \to a=5 \qquad \frac{a}{b} \to \frac{5}{7}$

عطف $x=\frac{-b}{3a} \to x=\frac{-a}{3} \to \frac{-a}{3}=-1 \to \boxed{a=3}$ $\qquad \frac{a}{b}=\frac{3}{5}$

$-4=-1+3-b-1 \to \boxed{b=5}$

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(9)

$f(x)=x^3+ax^2+bx+c$

$\xrightarrow{(0,4)} 0+0+0+c=4 \to \boxed{c=4}$

$f'(x)=3x^2+2ax+b \to 0+0+b=0 \to \boxed{b=0}$

$\to 3x^2+2ax \to x(3x+2a) \to 3x+2a=0 \to x=-\frac{2a}{3}$

$\left(\frac{-2a}{3}\right)^3+a\left(\frac{-2a}{3}\right)^2+4=0 \to \frac{-8a^3}{27}+a\left(\frac{4a^2}{9}\right)+4=0$

$\frac{-8a^3}{27}+\frac{12a^3}{27}=-4 \to 4a^3=\frac{-4\times 27}{4} \to a^3=-27 \to \boxed{a=-3}$

$f(x)=x^3-3x^2+4 \to f'(x)=3x^2-6x \to 3x(x-2)$; $0,\ 2$ ④

x: 0, 2 — $+$ | $-$ | $+$ (max, min) $\to$ طول مینیمم $= ②$

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(10)

مشتق اول $f'(x) \to 4x^3-12x=0 \to x=\pm\sqrt{3},\ 0$ $\xrightarrow{\text{تغییر علامت}}$ $-\sqrt{3}$, 0, $\sqrt{3}$: $-$ | $+$ | $-$ | $+$

مشتق دوم $f''(x) \to 12x^2-12=0 \to x=\pm 1$

$A: x=-\sqrt{3} \to f(-\sqrt{3})=9-18+5=-4 \to A\begin{vmatrix}-\sqrt{3}\\-4\end{vmatrix}$

$B: x=\sqrt{3} \to f(\sqrt{3})=9-18+5=-4 \to B\begin{vmatrix}\sqrt{3}\\-4\end{vmatrix}$ ②

$C: x=1 \to 1-6+5=0 \to C\begin{vmatrix}1\\0\end{vmatrix}$

$D: x=-1 \to 1-6+5=0 \to D\begin{vmatrix}-1\\0\end{vmatrix}$

چون شیب پاره‌خط‌های AB و CD برابر صفر است ($\frac{\Delta y}{\Delta x}=0$) نتیجه می‌گیریم که زاویه بین‌شان صفر است.