اهما ارشری

$f(x) = \cos^3(2x) + ax^2 + b \ , \ f'(x) = -3\times 2 \sin^2(2x) + 2ax$

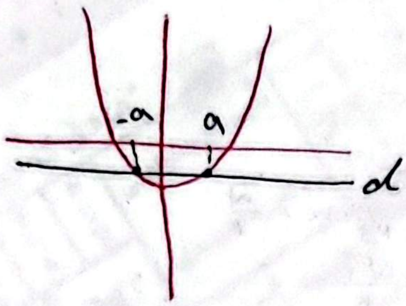
1

$\lim_{x\to 0} \frac{f(x)}{x} = \lim_{x\to 0} \frac{\cos^2 2x + ax^2 + b}{x} \longrightarrow \lim_{x\to 0} \frac{1x^2 + ax^2 + b}{x} = 0 \rightarrow b=0$

$\lim_{x\to 0} \frac{f'(x)}{x} = \lim_{x\to 0} \frac{-6\sin^2(2x) + 2ax}{x} \longrightarrow \lim_{x\to 0} \frac{-6\times 4x^2 + 2ax}{x} \rightarrow \lim_{x\to 0} \frac{-24x^2 + }{}$

$\longrightarrow \lim_{x\to 0} \rightarrow -24x + 2a = 2 \rightarrow \boxed{a=1}$ $\qquad a + b \Rightarrow 1 + 0 = 1$

2



$f'(x) = x^2 - 1$

$\hookrightarrow f'(x) = 2x$

$f'(a) \cdot f'(-a) = -1$

$\hookrightarrow 2a \times (-2a) = -1 \longrightarrow a^2 = 1/4 \longrightarrow a = \pm \frac{1}{2}$

$\xrightarrow{a>0} a = \frac{1}{2}$

$f(x) = x^2 - 1 \xrightarrow{a=1/2} 1/4 - 1 = \frac{-3}{4}$

$\quad \xrightarrow{a=-1/2} \frac{1}{4} - 1 \ , \ \frac{-3}{4} \longrightarrow -\frac{3}{4} \times 2 = \boxed{-1/5}$ جواب

3

$A \begin{vmatrix} 2/5 \\ 6 \end{vmatrix} \quad B \begin{vmatrix} -0/5 \\ -12 \end{vmatrix} \quad m_{AB} \frac{6-(-12)}{2/5-(-0/5)} = \frac{18}{3} = 6 \qquad y = 6x - 9$

$f(x) = \frac{a}{2x-1} \longrightarrow f'(x) = \frac{-2a}{(2x-1)^2} \longrightarrow f'(x) = \frac{-2a}{(4x^2+1-4x)} = 6$

$f(x) = \frac{a}{2x-1} = 6x - 9$

$\longrightarrow \begin{cases} a = -3(2x-1)^2 \\ a = (2x-1)(6x-9) \end{cases} \Rightarrow -3(2x-1) = 6x - 9 \longrightarrow -6x + 3 = 6x - 9$

$12x = 12 \longrightarrow \boxed{x=1}$

$a = -3 \longrightarrow f(0) = \frac{-3}{9} = -\frac{1}{3}$

④

$$f(x) = \frac{x+a}{ax+1} \longrightarrow f'(x) = \frac{1(ax+1) - (a)(x+a)}{(ax+1)^2}$$

$$\frac{\cancel{ax}+1-\cancel{ax}-a^2}{(ax+1)^2} = \frac{1-a^2}{(ax+1)^2} \longrightarrow y'(1) = \frac{1-a^2}{(a+1)^2} = 2 \longrightarrow 2(a^2+2a+1) = 1-a^2$$

$$2a^2+4a+2-1+a^2=0 \longrightarrow 3a^2+4a+1 \longrightarrow (a+1)(3a+1)=0 \begin{cases} a=-1 \ ✗ \\ a=-\frac{1}{3} \ ✓ \end{cases}$$

$$y = \frac{x-\frac{1}{3}}{-\frac{1}{3}x+1} \longrightarrow \frac{3x-1}{-x+3} \xrightarrow{x=1} \frac{3-1}{-1+3} = 1$$

$$1 = 2+b \longrightarrow b=-1 \longrightarrow a-b = -\frac{1}{3}+1 = \frac{2}{3}$$

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⑤

$$f(x) = g(x) \longrightarrow \frac{3}{2}\sin x = \sin x + \frac{1}{2}\cos x$$

$$\frac{1}{2}\sin x = \frac{1}{2}\cos x \longrightarrow \sin x = \cos x \xrightarrow[{[0,\frac{\pi}{2}]}]{} x = \frac{\pi}{4}$$

$$f'(x) = \cos x - \frac{1}{2}\sin x \longrightarrow f'(\frac{\pi}{4}) = \frac{2\sqrt{2}}{2\times 2} - \frac{\sqrt{2}}{4} = \frac{\sqrt{2}}{4}$$

$$f(\frac{\pi}{4}) = \frac{2\sqrt{2}}{2\times 4} + \frac{\sqrt{2}}{4} = \frac{3\sqrt{2}}{4} \longrightarrow y = \frac{\sqrt{2}}{4}x + b \xrightarrow{x=\frac{\pi}{4}} \frac{\sqrt{2}}{4}\times\frac{\pi}{4} + b = \frac{3\sqrt{2}}{4}$$

$$\longrightarrow b = -\frac{\pi\sqrt{2}}{16} + \frac{3\sqrt{2}}{4} \xrightarrow{y=0} \frac{\sqrt{2}}{4}x + \frac{3\sqrt{2}}{4} - \frac{\pi\sqrt{2}}{16} = 0 \longrightarrow x = \frac{\pi}{4}+3$$

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⑥

$$f(x) = 2x^3 - 3x^2 - 12x + 1 \longrightarrow f'(x) = 6x^2 - 6x - 12$$

$$\xrightarrow{\div 6} x^2 - x - 2 \longrightarrow (x-2)(x+1) \quad x=2,\ x=-1$$

	-1		2	
$+$		$-$		$+$
↗	8	↘	-19	↗

$$A\,|\,{}^{2}_{-19} \quad B\,|\,{}^{-1}_{8} \quad m_{AB} = \frac{8-(-19)}{-1-2} = \frac{27}{-3} = \boxed{-9}$$

$$y = -9x + \cancel{b}^{\,-1} \longrightarrow 8 = 9+b \longrightarrow b=-1$$

$$-9x - 1 = 2x^3 - 3x^2 - 12x + 1 \longrightarrow 2x^3 - 3x^2 - 3x + 2$$

$$\xrightarrow{x=-1} -2+3-3+2=0$$

	2	-3	-3	2
-1		-2	5	-2
	2	-5	2	0

$$(x+1)(\underbrace{2x^2-5x+2}_{\substack{x^2+5x+4 \\ (x+\frac{4}{2})(2x+1)}}) \longrightarrow (x+1)(x+2)(2x+1) \begin{cases} x=-1 \\ x=-2 \\ x=-\frac{1}{2} \end{cases}$$

$\boxed{\text{۳ نقطه}}$

(۷)

$y' = 3kx^2 + 2(k+1)x \xrightarrow{\text{مشتق}} y'' = 6kx + 2(k+1) = 0$

$x = \dfrac{-k-1}{3k}$

$\boxed{\text{حالت I}}$ $x < 0 \Rightarrow \dfrac{-k-1}{3k} < 0 \xrightarrow{\text{تعیین علامت}}$

-1		0	
$-$	$+$		$-$

$\longrightarrow$ x در بازه $\rightarrow (-\infty, -1) \cup (0, +\infty)$

$\boxed{\text{حالت II}}$ $y = k \times \left(\dfrac{-k-1}{3k}\right)^3 + (k+1)\left(\dfrac{-k-1}{3k}\right)^2 > 0 \longrightarrow \dfrac{-(k+1)^3}{9k^2} + \dfrac{-(k+1)^3}{27k}$

$\longrightarrow \dfrac{-k(k+1)^3}{27k^2} > 0 \xrightarrow{\text{تعیین علامت}}$

-1	مضاعف 0	
$+$	$-$	$-$

$\longrightarrow$ x در بازه $\rightarrow (-\infty, -1)$

اشتراک $I \cap II \longrightarrow x \in (-\infty, -1) \longrightarrow$ بعمار !

(8)

$f'(x) \to 3x^2 + 2ax + b \xrightarrow{x=-1} 3 - 2a + b = 0 \to b - 2a = -3$

$f(x) = x^3 + ax^2 + bx - 1 \xrightarrow{\left|\begin{smallmatrix}-1\\-4\end{smallmatrix}\right.} -1 + a - b - 1 = -4 \to a - b = -4 + 2$

$a - b = -2$

$$\begin{cases} \cancel{b} - 2a = -3 \to b - 10 = -3 \to b = 7 \\ a - \cancel{b} = -2 \end{cases}$$

$-a = -5 \to a = 5$

$\frac{a}{b} \to \boxed{\frac{5}{7}}$

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(9)

$f(x) = x^3 + ax^2 + bx + c$

$\xrightarrow{\left|\begin{smallmatrix}0\\4\end{smallmatrix}\right.} 0 + 0 + 0 + c = 4 \to \boxed{c = 4}$

$f'(x) = 3x^2 + 2ax + b \to 0 + 0 + b = 0 \to \boxed{b = 0}$

$\to 3x^2 + 2ax \to x(\underbrace{3x + 2a}) \to 3x + 2a = 0 \to x = -\frac{2a}{3}$

$\left(\frac{-2a}{3}\right)^3 + a\left(\frac{-2a}{3}\right)^2 + 4 = 0 \to \frac{-8a^3}{27} + a\left(\frac{4a^2}{9}\right) + 4 = 0$

$\frac{-8a^3}{27} + \frac{12a^3}{27} = -4 \to \cancel{4}a^3 = \frac{-4 \times 27}{\cancel{4}} \to a^3 = -27 \to \boxed{a = -3}$

$f(x) = x^3 - 3x^2 + 4 \to f'(x) = 3x^2 - 6x \to 3\underset{0}{x}(\underset{2}{x - 2})$

	0		2	
+		−		+
↗	Max	↘	Min	↗

$\longrightarrow$ طول min نسبی = ۲

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(10)

مشتق اول $f'(x) \to 4x^3 - 12x = 0 \to x = \pm\sqrt{3}, 0 \xrightarrow{\text{تغییر علامت}}$

	$-\sqrt{3}$		0		$\sqrt{3}$	
−		+		−		+
↘		↗		↘		↗

مشتق دوم $f''(x) \to 12x^2 - 12 = 0 \to x = \pm 1$

$A: x = -\sqrt{3} \to f(-\sqrt{3}) = 9 - 18 + 5 = -4 \to A\left|\begin{smallmatrix}-\sqrt{3}\\-4\end{smallmatrix}\right.$

$B: x = \sqrt{3} \to f(\sqrt{3}) = 9 - 18 + 5 = -4 \to B\left|\begin{smallmatrix}\sqrt{3}\\-4\end{smallmatrix}\right.$

$C: x = 1 \to 1 - 6 + 5 = 0 \to C\left|\begin{smallmatrix}1\\0\end{smallmatrix}\right.$

$D: x = -1 \to 1 - 6 + 5 = 0 \to D\left|\begin{smallmatrix}-1\\0\end{smallmatrix}\right.$

چون شیب پاره‌خط‌های AB و CD برابر صفر است $\left(\frac{\Delta y}{\Delta x} = 0\right)$ نتیجه می‌گیریم که زاویه بین‌شان $\theta = 0$ است