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مبنا رضایی

۱) $f'(x) = -4\sin 2x + 2a\cos x$

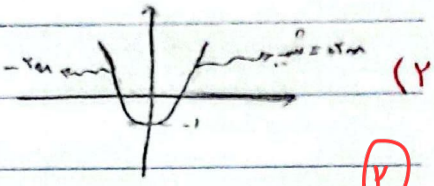
$\lim \frac{f'(x)}{x} = 2 \rightarrow \lim \frac{f'(x) - f'(0)}{x - 0} = \frac{f'(x)}{x} = f''(x) = -12\cos 2x + 2a$

$-12 + 2a = 2 \rightarrow a = 7$ (۲)

$\lim \frac{f(x) - f(0)}{x - 0} = 0 \rightarrow f(0) = 0 \rightarrow 1 + b = 0 \rightarrow b = -1$

$a + b = 6$ ✓

۲) ۲ عطاص برهم عمودند ← معکوس قرینه هستن



(۲)

$2m \rightarrow -\frac{1}{2m} = -2m \rightarrow 4m^2 = 1 \rightarrow m = \pm\frac{1}{2} \rightarrow y = -\frac{3}{2}$

مجموع عرض ها $= 2x - \frac{3}{2} = \boxed{-\frac{3}{2}}$ ✓

۳) شیب خط واصل ۲ نقطه $= \frac{18}{3} = 6$ = معادله خط $6m - 9$

$y = \frac{a}{2m-1} \rightarrow y' = \frac{-2a}{(2m-1)^2} = 6 \rightarrow 12m^2 - 12m + 3 = -a$ (I)

$\frac{a}{2m-1} = 6m - 9 \rightarrow 12m^2 - 24m + 9 = a$ (II) (۲)

(I), (II) $\rightarrow -12m^2 + 12m - 3 = 12m^2 - 24m + 9 \rightarrow 24m^2 - 36m + 12 = 0$

$m = 1$ ✓
$m = \frac{1}{2}$ ✗ غیرقابل قبول

$m = 1 \xrightarrow{(I)} a = -3$ ✓

$f(0) = \frac{-3}{2(0) - 1} = \boxed{-\frac{1}{3}}$

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(۲ $\frac{(am+1)-a(m+a)}{(am+1)^2} = 2 \xrightarrow{m=1} \frac{1-a^2}{(a+1)^2} = \frac{1-a}{a+1} = 2 \rightarrow 1-a = 2a+2 \rightarrow a = -\frac{1}{3}$ (۲

$(\pi) \rightarrow \frac{1+a}{a+1} = 2+b \rightarrow 1 = 2+b \rightarrow b = -1$ $\boxed{a-b = \frac{2}{3}}$ ✓

(۵ $f(m) = g(m) \rightarrow \frac{3}{2}\sin m = \sin m + \frac{1}{2}\cos m \rightarrow \sin m = \cos m$ $\xleftarrow{[0,\pi]}$ $m = \frac{\pi}{4}$

$f'(m) = \cos m - \frac{1}{2}\sin m \xrightarrow{m=\frac{\pi}{4}} \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{4} = \frac{\sqrt{2}}{4}$ (۲

$\frac{3\sqrt{2}}{4} = \frac{\sqrt{2}}{4} \times \frac{\pi}{4} \boxed{-\frac{\pi\sqrt{2}}{16} + \frac{3\sqrt{2}}{4}}$ $\frac{\sqrt{2}}{4}m + b$ ← خط مماس

b

$\frac{\sqrt{2}}{4}m = \frac{\pi\sqrt{2}}{16} - \frac{3\sqrt{2}}{4} \rightarrow \boxed{m = \frac{\pi}{4} - 3}$ ✓

(۷ $f'(m) = 4m^2 - 4m - 12 = 0 \rightarrow m = -1, 2$

	$-\infty$	-1	2	$+\infty$
f'		+	−	+
f		↗	↘	↗
		8	-19	

(۲

$m_{AB} = \frac{-27}{3} = -9$

$4m^2 - 4m - 12 = -9 \xrightarrow{\div 3} 2m^2 - 2m - 1 = 0$ ✓

$\Delta > 0 \rightarrow$ قبول

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(۷ نمره)

$y' = 3kx^2 + 2(k+1)x \longrightarrow y'' = 6kx + 2k + 2 = 0$

$y = kx^3 + (k+1)x^2 \to y' = 3kx^2 + 2(k+1)x \to y'' = 6kx + 2(k+1) = 0$

$(3x+2)k = -2 \longrightarrow k = -1, -2$

$x = \frac{-k-1}{3k}$ تقعر رو به پایین $\to -\frac{k+1}{3k} < 0 \to \frac{-1}{-|+|-} \to k<-1 \text{ , } k>0$ (I)

(I) ∩ (II) $\to k>0$

N قابل قبول

$\to -\frac{k+1}{3k}k + k + 1 > 0 \to -\frac{k+1}{3} + k + 1 > 0 \to \frac{2k+2}{3} > 0 \to k+1 > 0 \to k > -1$ (II)

تنها مقدار صحیح و مثبت است

(۸ نمره)

$y' = 3x^2 + 2ax + b \xrightarrow{(-1,-4)} 3 - 2a + b = -4 \to -2a + b = -7$

$x_{\text{می‌نیمم}} = -\frac{b}{3a} \to x = \frac{-a}{3} \to \frac{-a}{3} = -1 \to a = 3$

$\frac{a}{b} = \frac{3}{5}$ (۱ نمره)

$x^3 + ax^2 + bx - 1 \xrightarrow{(-1,-4)} -1 + a - b - 1 = -4 \to a - b = -2$

$-2 = -1 + 3 - b - 1 \to b = 5$

$\begin{cases} -2a + b = -7 \\ a - b = -2 \end{cases} \longrightarrow -a = -9 \to a = 9 \text{ , } b = 11$

$\frac{a}{b} = \frac{9}{11}$

(۹ نمره)

$y' = 3x^2 + 2ax + b \xrightarrow{(0,0)} b = 0$

$y = x^3 + ax^2 + c \xrightarrow{(0,4)} c = 4$

$\to x^3 + ax^2 + 4 = 0 \to \frac{-8a^3}{27} + \frac{4a^3}{9} + 4 = 0 \to a^3 = -27 \to a = -3$

$\to 3x + 2a = 0 \longrightarrow x = \frac{-2a}{3}$

$\to 3x + 6 = 0 \to x = -2$

x		0		$\frac{-2a}{3}$	
y'	$+$		$-$		$+$
y	↗		↘	min	↗

$\to f(-\frac{2a}{3}) = 0 \to (\frac{-2a}{3})^3 + a(-\frac{2a}{3})^2 + 4 = 0$

$\to \frac{-8a^3}{27} + \frac{4a^3}{9} + 4 = 0 \to a^3 = -27 \to a = -3$

$x = \frac{-2a}{3} \to x_{\text{min}} = \frac{-2(-3)}{3} = 2$

۱۵) $\sqrt{3}, -\sqrt{3}, 0$ ← $4x(x^2-3)=0$ ← $f'(x)=4x^3-12x$

$-\sqrt{3}$		0		$\sqrt{3}$	
−		+	−		+

نقاط A و B = $(\sqrt{3}, -4)$ ، $(-\sqrt{3}, -4)$ ← $x=0$

۲

$f''(x)=12x^2-12=0$ ← $12(x^2-1)=0$ ← $x=\pm 1$

نقاط C و D = $(1,0)$ و $(-1,0)$ ← $x'=0$

۲ خط سازی اندر خوردی ندارند

لبه زاویه بین آنها صفر است.